Basic Maths – XI (Mgmt.)

SET - A

Group A **[5×3×2 = 30]**

1. a. Rewrite $-6 \le x \le 1$ by using modulus sign. Ans: $|2x + 5| \le 7$

   b. If A = {1, 2, 4, 7}; find the relation on A satisfying the condition $x + y \ge 10$. Ans: {(4, 7), (7, 4), (7, 7)}

   c. Verify that: $\log (1 + 2 + 3) = \log 1 + \log 2 + \log 3$

2. a. Prove that: $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

   b. If A be A.M. and H be H.M. between two quantities a and b. Show that:

   $$\frac{a - A}{a - H} \times \frac{b - A}{b - H} = \frac{A}{H}$$

   c. Define symmetric and skew symmetric matrix with examples.

3. a. Solve the following equations using Cramer's rule:

   $5x - 2y + 2 = 0.$ $2x + 5y = 24.$ Ans: $\frac{38}{29}, \frac{124}{29}$

   b. If $x = \frac{a - ib}{a + ib}$, show that $x^2 + y^2 = 1$.

   c. For what value of k, the equation $(3k + 1)x^2 + 2(k + 1)x + k = 0$ has reciprocal roots? Ans: $-\frac{1}{2}$

4. a. Find the equation of straight line through (–1, –3) having equal intercepts on the axes? Ans: $x + y + 4 = 0$

   b. Find the equation of tangent to the circle $x^2 + y^2 = a^2$ which are parallel to $3x + 4y - 5 = 0$ Ans: $3x + 4y \pm 5a = 0$

   c. Evaluate: $\lim\limits_{\theta \to 0} \frac{\operatorname{cosec}\theta - \cot\theta}{\theta}$ Ans: $\frac{1}{2}$

5. a. Find $\frac{dy}{dx}$ of $x^2 + y^2 = a^2$. Ans: $\frac{-x}{y}$

   b. The side of a square sheet is increasing at the rate of 5 cm/min. At what rate is the area increasing when the side is 12 cm? Ans: 120 cm^2 /min

   c. Evaluate: $\int x^2 . e^{ax}\, dx$. Ans: $\frac{x^2.e^{ax}}{a} - \frac{2xe^{ax}}{a^2} + \frac{2e^{ax}}{a^3} + c$

Group B [5×2×4 = 40]

6. a. If A, B and C are the subsets of a universal set U, prove that
$A - (B \cup C) = (A - B) \cap (A - C)$

OR

Define the term tautology. Prepare truth table of the statement
$(p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$ and verify that it is a tautology.

b. Using different characteristic, sketch the graph of the function
$x = \log_3 y$.

7. a. If $\dfrac{\sin(A - B)}{\sin(A + B)} = \dfrac{a^2 - b^2}{a^2 + b^2}$, prove that triangle ABC is either isosceles or right angled.

OR

If $B = 45°$, $a = 3 + \sqrt{3}$, $b = 2\sqrt{3}$, solve the triangle.

Ans: $A = 75°$, $C = 60°$, $c = 3\sqrt{2}$

b. Without expanding the determinant, show that

$$\begin{vmatrix} a^2 & 2ab & b^2 \\ b^2 & a^2 & 2ab \\ 2ab & b^2 & a^2 \end{vmatrix} = (a^3 + b^3)^2$$

8. a. Using matrix method or Cramer's rule, solve the following system:
$2x - y + z = -1$, $x - 2y + 3z = 4$, $4x + y + 2z = 4$ Ans: –1, 2, 3

b. If the equation $x^2 + px + q = 0$ and $x^2 + qx + p$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$.

9. a. Show that the tangent to the circle $x^2 + y^2 = 100$ at the points (6, 8) and (8, –6) are perpendicular to each other.

b. Evaluate: $\displaystyle\lim_{x \to \theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$ Ans: $\sin\theta - \theta \cos x$

OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} x^2 + 2 & \text{for } x < 5 \\ 20 & \text{for } x = 5 \\ 3x + 12 & \text{for } x > 5 \end{cases}$$

Show that f(x) has removable discontinuity at x = 5.

10. a. Find from the definition the derivative of $\sqrt{5x + 3}$.

b. Find the area of the region between the curves $y = x^2$ and $x = y^2$. Ans: $\frac{1}{3}$

Group C [5×6 = 30]

11. Define one to one function and onto function. Let a function $f : A \rightarrow B$ be defined by $f(x) = x^2$ with $A = \{-2, -1, 0, 1, 2\}$ and $B = \{0, \frac{1}{6}, \frac{2}{3}\}$.

Find the range of f. Is the function f one to one and onto both? Give reason.

12. Find the sum to n terms of the series:

$$1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots.. \qquad \text{Ans: } \frac{35}{16} - \frac{12n+7}{16.5^{n-1}}$$

OR

Prove that: $1^2 + 2^2 + 3^2 + \ldots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

13. Find the angle between the line pair represented by $ax^2 + 2hxy + by^2 = 0$. Also, find the condition of perpendicularity.

14. Define conjugate of a complex number. Find the square root of $\dfrac{2-36i}{2+3i}$.

Ans: $\pm (1 - 3i)$

15. Show that the rectangle of largest possible area, for a given perimeter is a square.

OR

Water flows into inverted conical tank at the rate of 42 cm^3/sec, when the depth of water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. Ans: $\dfrac{21}{8\pi}$ cm/S.

Set - II

Group A **(5×3×2 = 30)**

1. (a) Construct the truth table for $\sim [P \vee (\sim q)$ [Ans: FFTF]
   (b) Let A = {1, 2, 3,4} and B = {1,3 5}, find the relation R from set A to set B determined by the condition x > y.
   [Ans: {3, 1). (4,1), (4,3)}]
   (c) Define even and odd function. Determine whether the function $f(x) = 2^x + 2^{-x}$ is even, odd or neither.
2. (a) If the sides of a triangle are $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$ then prove that the greatest angle is 120°.
   (b) Find three numbers in A.P whose sum is 21 and product is 315.
   [Ans: 5,7,9 or 9,7,5]
   (c) If $A = \begin{pmatrix} 2 & -2 \\ -3 & 4 \end{pmatrix}$ be a matrix, show that $A^2 - 6A + 2I = 0$ where I and 0 are 2 ×2 identity and null matrices respectively.
3. (a) Solve the followings system of linear equations, if possible:
   $5x - 3y = 9$, $10x - 6y = 16$.
   (b) Find the value of x and y if $(x + 2) + yi = (3 + i)(1 + 2i)$
   [Ans: –1, 7]

(c) Determine the nature of the roots of the equation $2x^2 + 3x - 2 = 0$

4. (a) Show that the points (1,2) and (2, –3) lie on opposite side of the line $5x - 2y - 3 = 0$.

(b) If one end of a diameter of the circle $x^2 + y^2 - 10x - 12y + 43 = 0$ be (2, 3) , find the other end. [Ans: (8,9)]

(c) Evaluate: $\lim_{x \to 0} \frac{a^x + b^x - 2}{x}$ [Ans: log ab]

5. (a) Find the derivative of $e^{5x}. \log x$. [Ans: $e^{5x} (\frac{1}{x} + 5 \log x)$]

(b) Find where the graph of the function $f(x) = 2x^3 - 6x^2 + 5$ is concave upward or concave downward.

(c) Evaluate: $\int \frac{x + 2}{\sqrt{x+1}}$ dx. [Ans: $\frac{2}{3} (x + 1)^{3/2} + 2 (x + 1)^{1\backslash 2} + c$]

Group B **(5× 2 × 4 = 40)**

6. (a) Define difference of two sets. If A, B and c are three non–empty sets then verify $A - (B \cap c) = (A - B) \cup (A - C)$

OR Let A = (–1, 4) and B = [3, 5) . Find $A \cup B$, $A \cap B$, A –B and B–A. [Ans: (–1, 5) , [3, 4). (–1, 3) [4, 5)]

(b) Draw the graph of y = cosx $\left(-\frac{\pi}{2} \le x \le \frac{\pi}{2}\right)$ using different characteristics.

7. (a) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1} z = \frac{\pi}{2}$, then show that $x^2 + y^2 + z^2 + 2xyz = 1$.

OR

If $8 R^2 = a^2 + b^2 + c^2$, prove that the triangle is right angled.

(b) Prove that: $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + 1\right)$

8. (a) Using row equivalent matrix method or inverse matrix method, solve the following equations.

$x + 4y + z = 18$, $3 x + 3y - 2z = 2$, $-4y + z = -7$ [Ans: 1, 3, 7]

(b) If the ratio of roots $x^2 + lx + m = 0$ is equal to the ratio of roots of $x^2 + px + q = 0$ then show that $p^2 m = l^2 q$.

9. (a) Find the condition that the line $l x + m y + n = 0$ may be tangent to the circle $x^2 + y^2 + 2gx + 2 fy + c = 0$.

[Ans: $(g^2 + f^2 - c) (l^2 + m^2) = (ln - lg - mf)^2$

(b) Evaluate: $\lim_{x \to \theta} \dfrac{x\cos\theta - \theta\cos x}{x - \theta}$ [Ans: $\cos\theta + \theta\sin\theta$]

OR

A function f (x) is defined as follows:

$$f(x) = \begin{cases} 3 + 2x & \text{for } \frac{-3}{2} \le x < 0 \\ 3 - 2x & \text{for } 0 \le x < \frac{3}{2} \\ -3 - 2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

Show that f (x) is continuous at x = 0 but discontinuous at $x = \frac{3}{2}$.

10. (b) Find the derivative of $\sqrt{3-2x}$ from first principle. [Ans: $\dfrac{-1}{\sqrt{3-2x}}$]

(b) Find area between the curves $y^2 = 4ax$ and $x^2 = 4ay$. [Ans: $\dfrac{16}{3} a^2$]

Group – C **(5× 6 = 30)**

11. Define domain and range of a function. Find domain and range of :

$y = f(x) = \sqrt{6 - x - x^2}$. Ans: [–3,2], [0, $\frac{5}{2}$]

12. The sum of three numbers in A.P. is 36. When the numbers are increased by 1, 4, 43 respectively, the resulting numbers are in G. P. Find the numbers. [Ans: 3, 12, 21 or 63,12 – 39]

13. Find the angle between two lines given by $y = m_1 x + c_1$ and $y = m_2 x + c_2$. Also state the condition for them to be perpendicular and parallel.

OR

Show that the homogeneous equation of second degree represents a line pair. Also find the equation of the straight lines through the origin and at right angles to the lines $x^2 - 5xy + 4y^2 = 0$ [Ans: $4x^2 + 5xy + y^2 = 0$]

14. State and prove De-moivré's theorem. Use it to compute

$(\cos 60° + i \sin 60°)^7$. [Ans: $\frac{1}{2} + i\sqrt{\frac{3}{2}}$]

15. A window is in the form of a rectangular surmounted by a semi- circle. If the total perimeter is 9 m; find the radius of semi- circle for the greatest window area. [Ans: $\dfrac{9}{4 + \pi}$ m.]

OR

A man of height 1.5m walks away from a lamp post of height 4.5m at the rate of 20 cm /s. How fast is the shadow lengthening when the man is 42 cm from the post? [Ans: 10 cm /s.]

Set - III

Group A **(5×3×2 = 30)**

1. (a) For any two real numbers x and y, prove that $|x + y| \le |x| + |y|$.
   (b) Prove that the function $f : R \to R$ defined by $f(x) = 3x - 1$ is one – one
   (c) Show that $f(x) = 7x - 5$ is increasing for all $x \in R$.
2. (a) Show that $\tan^{-1} x = \text{Sin}^{-1} \frac{2x}{1+x^2}$
   (b) Find the sum to infinity of the series:
   $3 + 5x + 7x^2 + \ldots\ldots\ldots\ldots,\ |x| < 1.$ [Ans: $\frac{3-x}{(1-x)^2}$]
   (c) If $A = \begin{pmatrix} 4 & -5 \\ 3 & 6 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$, find $(AB)^T$. [Ans: $\begin{pmatrix} 13 & 0 \\ 22 & -3 \end{pmatrix}$]
3. (a) Solve by cramer's rule: $2x-y = 5$, $x-2y = 1$ [Ans: 3, 1]
   (b) Find square root of $-7+ 24i$ [Ans: $\pm (3 + 4i)$]
   (c) What value of k will the equation $x^2 - 2kx + 7k-12 = 0$ have equal roots? [Ans: 3 or 4]
4. (a) Find the value of a so that the lines $3x + y = 2$, $ax + 2y = 3$ and $2x - y = 3$ are concurrent. [Ans: 5]
   (b) Find the equation of the circle with center at (a, b) and touching Y–axis. [Ans: $x^2 + y^2 - 2ax - 2by + b^2 = 0$]
   (c) Evaluate: $\lim\limits_{x\to 0} \frac{\sin x^\circ}{x}$ [Ans: $\frac{\pi}{180}$]
5. (a) Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3axy = 0$. [Ans: $\frac{ay-x^2}{y^2-ax}$]
   (b) Examine whether the function $f(x) = 15x^2 - 14x + 1$ is increasing or decreasing at $x = \frac{2}{5}$ and $x = \frac{5}{2}$.
   (c) Integrate: $\int \text{Sin}^2 2x\, dx$ [Ans: $\frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) + c$]

Group B **(5×2×4 = 40)**

6. (a) Define the complement of a set. State and prove De Morgan's laws on set theory.

   OR

   Define the term contradiction. If p and q be two statements, prove that $\sim (p \vee q) \wedge q$ is a contradiction.
   (b) Sketch the graph of $y = (x + 1)(x - 2)(x - 3)$ with different characteristics.
7. (a) Solve for general values of x : $\cos x + \cos 2x + \cos 3x = 0$

[Ans: $(4n \pm 1)\frac{\pi}{4}$, $(6n \pm 2)\frac{\pi}{3}$]

OR

If $8R^2 = a^2 + b^2 + c^2$. Prove that the triangle is right angled.

(b) Prove that: $\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$

8. (a) Using Row equivalent matrix method or inverse matrix method, solve the equations:

$3x + 5y = 2$, $2x - 3z = -7$, $4y + 2z = 2$ [Ans: 4, –2, 5]

(b) Prove that the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ cannot have more than two roots.

9. (a) Prove that the line $y = mx + c$ is tangent to the circle $x^2 + y^2 = a^2$ if $c = \pm a\sqrt{1 + m^2}$. Also, show that $3x + 4y = 20$ is tangent to the circle $x^2 + y^2 = 16$.

(b) Evaluate: $\lim_{x \to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$ [Ans: $\cot\theta + \theta\,\mathrm{cosec}^2\,\theta$]

OR

A function f(x) is defined as $f(x) = \begin{cases} 2x - 3 & \text{for } x < 2 \\ 1 & \text{for } x = 2 \\ 3x - 5 & \text{for } x > 2 \end{cases}$

Is f(x) continuous at x = 2. If not, how can f (x) be made continuous at x = 2.

10. (a) Find, from first principle, the differential coefficient of $\sqrt{\sec x}$

[Ans: $\frac{1}{2}\sqrt{\sec x}.\tan x$]

(b) Find the area between the curve $y^2 = 16x$ and line $y = 2x$. [Ans: $\frac{16}{3}$]

Group C (5× 6 = 30)

11. Define inverse function. If $f : R \to R$ be defined by $f(x) = x^2 - 3$, find $f^{-1}(x)$, Also show that $fof^{-1}(x) = f^{-1}of(x)$. [Ans: $\sqrt{x + 3}$]

12. Using mathematical induction, prove that $1.3 + 2.4 + 3.5 + \ldots\ldots\ldots\ldots + n(n + 2) = \frac{1}{6}n(n + 1)(2n + 7)$, for any positive integer n.

13. State De- moivre's theorem and using this theorem solve $z^6 = 1$.

14. Derive the formula for the length of perpendicular from a point (x_1, y_1) to a line $x\cos\alpha + y\sin\alpha = p$. Also, find the distance between the parallel lines $5x + 12y + 8 = 0$ and $10x + 24y - 3 = 0$ [Ans: $\frac{19}{16}$]

OR

Prove that the straight lines joining origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angle if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$.

15. Find the local maxima and minima, and also the point of inflection (if exists) of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval $(-1, 2)$. Also examine whether the function is increasing or decreasing at $x = 0$

[Ans: Max value $\frac{7}{2}$ at $x = -\frac{1}{2}$, Min. Value $\frac{-25}{2}$ at x= 3\2, point of inflection $\frac{1}{2}$]

OR

A 5 m ladder leans against a vertical wall. If the top slides downwards at the rate of 12 m\min; find the speed of the lower end when it is 4m from the wall. [Ans: 9m/min]

Set - IV

Group A **(5×3 × 2 = 30)**

1. (a) Let $A = [-2, 4)$ and $B = (2, 5]$, compute $A \cap B$ and $A - B$,

[Ans: (2, 4), [–2,2].]

(b) Prove that : $\log_a \sqrt{a\sqrt{a\sqrt{a^2}}} = 1$

(c) If $f(x) = \begin{cases} 2 - x & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

then find $\frac{f(h) - f(1)}{h}$ when $h \geq 0$. [Ans: $\frac{1-h}{h}$]

2. (a) Solve: $\sin x + \sqrt{3} \cos x = \sqrt{2}$ [Ans: $2n\pi + \frac{\pi}{6} \pm \frac{\pi}{4}$]

(b) If a, b, c, are in A. P., a, x, b, are in A.P. and b, y, c are in A.P. prove that $\frac{1}{x} + \frac{1}{y} = \frac{2}{b}$.

(c) If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 0 \\ -5 & 2 \end{pmatrix}$, verify that $(AB)^T = B^{T\cdot} A^T$

3. (a) Solve the equations using cramer's rule. $3x - 2y = 8$, $5x + 3y = 7$
[Ans: 2, – 1]

(b) If $\alpha = \frac{-1 + \sqrt{-3}}{2}$, $B = \frac{-1 - \sqrt{-3}}{2}$, Show that $\alpha^4 + \alpha^2 \beta^2 + \beta^4 = 0$.

(c) Find the value of K so that the sum of the roots of $3x^2 + (3K + 1) x - (k + 5) = 0$ is equal to the product of the roots. [Ans: 2]

4. (a) Determine the equation of straight line the portion of which intercepted by the axes, is divided by the points (–5, 4) in the ratio 1:2. [Ans: $8x - 5y + 60 = 0$]

(b) Find the equation of the circle with centre at (4, –1) and passing through origin. [Ans: $x^2 + y^2 - 8x + 2y = 0$]

(c) Evaluate: $\lim_{x\to\infty} (\sqrt{x} - \sqrt{x-3})$. [Ans: 0]

5. (a) Find the derivative of. $Sec^2 (\tan\sqrt{x})$

[Ans: $\frac{1}{\sqrt{x}} . Sec^2 (\tan \sqrt{x}) . \tan (\tan\sqrt{x}) \sec^2 \sqrt{x}$]

(b) Find the interval in which the function $f(x) = 2x^3 - 15x^2 - 36x + 1$ is increasing or decreasing. [Ans: increasing on $(-\infty, -1) \cup (6, \infty)$ and decreasing on $(-1,6)$]

(c) Evaluate : $\int \frac{x}{(1-x^2)^{3/2}} dx$. [Ans: $\frac{1}{\sqrt{1-x^2}} + c$]

Group B **[5× 2× 4 = 40]**

6. (a) Define symmetric difference between two sets. If A and B are any two sets, prove that

$A \Delta B = (A \cup B) - (A \cap B)$

OR

For any real number x and positive number a, show

$|x| < a \Leftrightarrow -a < x < a$.

(b) Draw graph of function $y = f(x) = -x^2 + 4x - 3$ with its characteristics.

7. (a) If $\tan^{-1}x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, then verify that $xy + yz + zx = 1$

OR

State and prove the cosine law for a triangle.

(b) Prove that : $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$

8. (a) Using row equivalent matrix method or inverse matrix method, solve the equations

$2x - y + 4z = -3$

$x - 4z = 5$

$6x - y + 2z = 10$ [Ans: $3, 7, -\frac{1}{2}$]

(b) If the equation $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, prove that either $a + b + c = 0$ or $a = b = c$.

9. (a) Prove that the circle $x^2 + y^2 + 2ax + c^2 = 0$ and $x^2 + y^2 + 2by + c^2 = 0$ touch if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{c^2}$

(b) Prove geometrically that $\lim_{x \to o} \frac{\sin x}{x} = 1$, where x is measured in radian,

OR

A function f(x) is defined as $f(x) = \begin{cases} 2-x^2 & \text{for } x < 2 \\ 2 & \text{for } x = 2 \\ x - 4 & \text{for } x > 2 \end{cases}$.

Is the function f(x) continuous at x = 2 ? If not, what type of discontinuity is this? How can the function be made continuous at x = 2 ?

10. (a) Find, from first principle, the derivative of $x + \frac{1}{\sqrt{x}}$ [Ans: $1 - \frac{1}{2x^{3/2}}$]

(b) Using integration, find the area of the circle $x^2 + y^2 = a^2$ [Ans: πa^2]

Group – C **(5×6 = 30)**

11. Define domain and range of a function. Find domain and range of $y = f(x) = \sqrt{21 - 4x - x^2}$ [Ans: D(f) = [–7,3], R(f) = [0, 5]

12. If a^2, b^2 c^2 are in A.P. prove that b + c, c+ a, a + b are in H.P.

OR

Define A. M. and G. M. If the A.M. Of two positive numbers a and b be twice the G.M., Show that a: b = $(2 + \sqrt{3}) : (2 - \sqrt{3})$.

13. Prove that the straight line joining origin to the points of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angled if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$

14. Find the cube roots of unity by using De-Moivre's theorem . Prove any three properties of cube roots of unity. [Ans: $\frac{-1 \pm i\sqrt{3}}{2}$, 1]

15. Test whether the function $f(x) = 2x^2 - 4x + 3$ is increasing or decreasing on the interval (1, 4]. Also show that the maximum area enclosed by 256 m fencing material of a rectangular garden is square. Also find its area. [Ans: $4096m^2$]

OR

Two concentric circles are expanding in a such a way that the radius of the inner circle is increasing at the rate of 8 cm /sec and that of the outer circle at the rate of 7 cm /sec. At a certain time, the radii of inner and

outer circles are respectively 24 cm and 30 cm. At what rate does the area between two circles change? [Ans: $60 \pi \ cm^2$ /sec]

Set - V

Group A (5×3×2 = 30)

1. (a) Define complement of a set. Prove that $\overline{A \cup B} = \bar{A} \cap \bar{B}$
   (b) If A = {1, 2, 3}, find the relation on A×A given by x + y <4. Is this relation a function? Give reason. [Ans: {(1, 1), (1, 2) , (2 , 1) }
   (c) Test the periodicity of the function f(x)= Sin2x and find its period. [Ans: periodic in π]

2. (a) Solve: cotx + tanx = 2 ($0 \leq x \leq \pi$.) [Ans: $\frac{\pi}{4}$]
   (b) Using principle of induction method, verify 3^{2n}-1 is divisible by 8.
   (c) If A = $\begin{bmatrix} 1 & 7 \\ 4 & 9 \end{bmatrix}$ then find the value of $A^T \cdot A$. [Ans: $\begin{pmatrix} 17 & 42 \\ 42 & 85 \end{pmatrix}$]

3. (a) Use cramer's rule to solve $\frac{2}{x} + \frac{3}{y} = 2$ and $\frac{8}{x} + \frac{9}{y} = 7$ [Ans: 2, 3]
   (b) If $x - iy = \sqrt{\frac{1-i}{1+i}}$, prove that $x^2 + y^2 = 1$
   (c) Show that the roots of the equation $x^2 - 4abx + (a^2 + 2b^2)^2 = 0$ are imaginary.

4. (a) Find the distance between two parallel lines 3x + 5y = 11 and 3x + 5y = – 23. [Ans: $\sqrt{34}$]
   (b) Find the equation of the circle concentric with $x^2 + y^2 - 8x + 12y + 15 = 0$ and passing through (5,4). [Ans: $x^2 + y^2 - 8x + 12y - 49 = 0$]
   (c) Find the limit of $f(x) = \frac{x^2 - 4}{x-2}$ as $x \to 2$. Is f(x) continuous? If not, find the point of discontinuity.
   [Ans: 4, 2]

5. (a) Find $\frac{dy}{dx}$ if $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ [Ans: $\frac{2}{1+x^2}$]
   (b) Evaluate: $\int \frac{1}{\sqrt{2x-3}-\sqrt{2x-4}} dx$ [Ans: $\frac{1}{3}$ [$(2x\text{-}3)^{3/2}$ $(2x - 4)]^{3/2}$ + c
   (c) Check the concavity of the function given by $f(x) = 7x^3 - 4x^2 + 9x + 10$.

Group B (5× 2×4 = 40)

6. (a) Define absolute value of a real number. For any two real numbers x and y, prove that $|x + y| \le |x| + |y|$

OR

If p and q are any two statements, show that

(i) $PVq \equiv q \vee p$

(ii) $\sim(p \vee q) \equiv (\sim p) \wedge (\sim q)$

(b) Presenting different characteristic, sketch the graph of function $y = 2^{-x}$

7. (a) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $x^2+y^2 + z^2 + 2xyz = 1$.

OR

State and prove "sine law"

(b) Show that: $\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

8. (a) Applying row equivalent method or inverse matrix method, Solve the equations.

$x - 3y - 7z = 6, \quad 2x + 3y + z = 9, \quad 4x + y = 7$

[Ans: 1, 3,–2]

(b) Show that the root of equation $(a^2 - bc)x^2 + 2(b^2 - ca)x + (c^2 - ab) = 0$ will be equal if either $b = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$

9. (a) Find the equation of the chord of the circle $x^2 + y^2 = 9$ Which is bisected at the point (–1,2). [Ans: 2y = x + 5]

(b) Prove geometrically that $\lim_{\theta \to o} \frac{\sin\theta}{\theta} = 1$ where θ is a radian.

OR

Define continuity of a function at a point. A function is defined as follows:

$$f(x) = \begin{cases} \dfrac{2x^2 - 18}{x - 3} & \text{for } x \neq 3 \\ 2k & \text{for } x = 3 \end{cases}$$

Ans: 12

Find the value of k so that f(x) is continuous at x = 3.

10. a. Find, from first principle, the derivative of $\sqrt{\sin 2x}$.

b. Find the area of ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Ans: 6 π sq. unit

Group – C **(5×6 = 30)**

11. Define bijective function. Prove that a function $f : R \to R$ defined by $f(x) = 3x - 1$ is bijective. Also, find f^{-1}. [Ans: $\frac{x+1}{3}$]

12. Show that A.M., G.M, and H.M. between any two unequal positive numbers satisfy the following relations
    a. $(G.M.)^2 = A.M. \times H.M.$
    b. A.M. > G.M. > H. M.

13. Define absolute value of complex number. Find the square roots of $\frac{8-15i}{i}$ [Ans: $\pm (1 - 4i)$]

14. Find the equations of the bisectors of the angles between the lines $3x - 2y + 1 = 0$ and $18x + y - 5 = 0$. Also identify the bisector of the acute angle. [Ans: $11x - 3y = 0$, $3x + 11y - 10 = 0$, $11x - 3y = 0$ (acute angle)]

OR

Prove that the line pair joining origin to the point of intersection of straight line $\frac{x}{h} + \frac{y}{k} = 2$ and the curve $(x - h)^2 + (y - k)^2 = a^2$ are perpendicular to each other if $h^2 + k^2 = a^2$.

15. List the criteria for the function $y = f(x)$ to have local maxima and local minima at a point. Find local maxima and local minima of the function $(f)x = 4x^3 - 15x^2 + 12x + 7$. Also, find point of inflection.
[Ans: Max. value $\frac{39}{4}$ at $x = \frac{1}{2}$, Min. value 3 at $x = 2$, point of inflection at $x = \frac{5}{4}$]

OR

Water flows into inverted conical tank at the rate of 42 cm^3/sec, when the depth of water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm. Ans: $\frac{21}{8\pi}$ cm/S.