Basic Mathematics - XII (Mgmt.)

SET 1

Grade: XII | Subject: Basic Mathematics | F.M.:100

Time: 3 hrs. | P.M.: 40

Model 1

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions.

Group 'A'

1. a) How many even numbers of 3 digits can be formed if the repetition of digits is allowed? [2]

[Ans:450**]**

b) Find the term independent of x in the expansion of

$$\left(x^2 - \frac{2}{x^3}\right)^{15}$$ [2]

[Ans: $t_7 = 320320$**]**

c) Let $G = \{1, -1, i, -i\}$ and the operation be of multiplication. Show that G is closed and associative under multiplication. [2]

2. a) Find the equation of the parabola whose vertex is at (3, 2) and the focus is at (5, 2). [2]

[Ans: $y^2 - 4y - 8x + 28 = 0$**]**

b) Find the ratio in which the line joining the points (–2, 4, 7) and (3, –5, –8) is divided by the xy-plane. [2]

[Ans:7:8**]**

c) Show that the given vectors are collinear $\vec{i} + 2\vec{j} + 4\vec{k}$, $2\vec{i} + 5\vec{j} - \vec{k}$, $3\vec{i} + 8\vec{j} - 6\vec{k}$ [2]

3. a) Differentiate: $2\tanh^{-1}\left(\tan\frac{x}{2}\right)$ [2]

[Ans: sec x**]**

b) Evaluate: $\lim_{x\to 0} \frac{x - \tan x}{x^3}$ [2]

[Ans: $-\frac{1}{3}$ **]**

c) By using partial fraction; integrate $\int \frac{13}{(3x+4)(4x+1)}dx$. [2]

[Ans: $\log(4x+1)-\log(3x+4)+C$ **]**

4. a) Solve the given differential equation; $2xy\,dx - x^2 dy = 0$ [2]

[Ans: $x^2 = cy$ **]**

b) A frequency distribution gives the following results (i) C.V = 5 (ii) $\sigma = 2$ (iii) Karl Pearson's coefficients of skewness = 0.5. Find the mean and mode of the distribution. [2]

[Ans: $\bar{x} = 40; M_0 = 39$ **]**

c) The chance that A can solve a certain problem is $\frac{1}{4}$ and the chance that B can solve it is $\frac{2}{3}$. Find the chance that the problem be solved if both try. [2]

[Ans: $\frac{3}{4}$ **]**

5. a) A committee of 5 is to be formed from 6 gentlemen and 4 ladies. In how many ways can it be done when
   i) At least two ladies are included?
   ii) At most two ladies are included? [4]

[Ans:186, 186**]**

b) Define a group. Let $(G,*)$ be a group then show that $(a*b)^{-1} = b^{-1} * a^{-1}$ for $a,b \in G$. [4]

6. a) If three successive coefficients in the expansion of $(1+x)^n$ are 28, 56 and 70, find n. [4]

[Ans: n = 8**]**

OR

Show that $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots = \frac{e}{2}$

b) Define parabola. Find the equation of Parabola in standard form. [4]

[Ans: $y^2 = 4ax$**]**

Find eccentricity, the co-ordinates of the centre and the foci of the following ellipses. $9x^2 + 5y^2 - 30y = 0$

[Ans: $\frac{2}{3};(0,3);(0,5)\text{and}(0,1)$]

7. a) Find, from first principles, the derivative of: $\log\left(\sin\frac{x}{a}\right)$ [4]

[Ans: $\left[\frac{1}{a}\cot\frac{x}{a}\right]$]

OR

Verify Rolle's theorem for $f(x)=\sin 2x$ in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

b) Evaluate: $\int\frac{dx}{2+3\cos x}$ [4]

[Ans: $\left[\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5}+\tan\frac{x}{2}}{\sqrt{5}-\tan\frac{x}{2}}\right)\right]+C$]

8. a) Solve: $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$ [4]

[Ans: $\sin\left(\frac{y}{x}\right)=Cx$]

b) State and prove theorem of total probability. [4]

9. Define dot product of a two vector, if θ be the angle between two vector. Interpret it geometrically. Also prove by using vector method that; $b^2=c^2+a^2-2ac\cos B$. [6]

OR

Define cross product of a two vector, if θ be the angle between two vector. Interpret it geometrically. Also prove by using vector method that $Sin(A+B)=SinACosB+CosASinB$.

10. Reduce the general equation of plane to the normal form. Also find the length of perpendicular from a given point on a given plane in normal form. [6]

11. From the following data between the ages of husbands and wife's. Calculate the two regression equations and find the husband's age when wife's age is 20 and wife's age when husband's age is 30. [6]

Wife's age (X)	18	20	22	23	27	28	30
Husband's age (Y)	23	25	27	30	32	31	35

[Ans: $b_{yx} = 0.915;\ b_{xy} = 1.02$**]**

Group 'C'

16. a) Convert the decimal numerals 86.25 into octal form. [2]

[Ans: 126.2_8**]**

b) Draw the graph of following inequalities and find the feasible region. [2]
$x + y \leq 6$, $x - y \geq -2$, $x \geq 0$, $y \geq 0$

c) Use the Trapezoidal Rule to approximate the integral $\int_1^2 \frac{1}{x}\,dx$. Find the error for the approximation. [2]

[Ans: –0.056853**]**

17. a) Using Newton - Raphson's method to find the positive root of $x^3 - 18 = 0$ in (2, 3). [4]

[Ans:2.620**]**

b) Solve the given equations by Gauss elimination method: [4]
$3x - y + z = -2$; $x + 5y + 2z = 6$; $2x + 3y + z = 0$

[Ans: $x = -2;\ y = 0;\ z = 4$**]**

OR

Use the Gauss-Seidel method to solve the system:

$$3x + y = 5$$

$$x - 3y = 5$$

[Ans: $x = 2;\ y = -1$**]**

18. Using simplex method, maximize $Z = 2x + 3y.$ [6]
Subject to

$$x + 2y \leq 10$$

$$2x + y \leq 14$$

$$x \geq 0,\ y \geq 0$$

[Ans: Max value Z=18 at (6, 2)**]**

OR

Using simplex method, minimize $W = 3x + 2y$
Subject to

$$2x + y \geq 6$$

$$x + y \geq 4$$

$$x \geq 0,\ y \geq 0$$

[Ans: W=10 at (2, 2)**]**

19. Evaluate, using Simpson's rule the integral $\int_0^1 \frac{dx}{1+x}$. Estimate the error in using the approximation $n=4$. [6]

[Ans: 0.69325; Error = 0.00052**]**

SET 2

Group 'A'

1. a) How many numbers between 3000 and 6000 be formed with the integers 1, 2, 3, 5, 6, 7 ? [2]

[Ans:120**]**

b) Find the middle term in the expansion of $\left(\frac{x}{a}-\frac{a}{x}\right)^{2n+1}$ [2]

[Ans: $(-1)^n \frac{(2n+1)!}{(n+1)!n!}\left(\frac{x}{a}\right); (-1)^{n+1} \frac{(2n+1)!}{n!(n+1)!}\left(\frac{a}{x}\right)$**]**

c) Let $(G,0)$ is a group, then the group equation $x o x = x$ has a unique solution $x = e$. [2]

2. a) Prove that the line $lx + my + n = 0$ touches the parabola $y^2 = 4ax$ if $ln = am^2$. [2]

b) Find the projection of the join of the pair of points (3, –1, 2) and (5, –7, 4) on the given line joining the points (0, 1, 0) and (1, 3, 7). [2]

[Ans: $\frac{4}{3\sqrt{6}}$ **]**

c) Find the area of a parallelogram whose adjacent side are determined by vectors $\vec{i}+2\vec{j}+2\vec{k}$ and $2\vec{i}-3\vec{j}+\vec{k}$. [2]

[Ans: $\sqrt{122}$ sq. units**]**

3. a) Differentiate: $\tanh^{-1}\left(\tan\frac{x}{2}\right)$ w. r. t. '*x*' . [2]

[Ans: $\frac{1}{2} Secx$ **]**

b) Prove that $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x+\sqrt{x^2+a^2}\right| + C$ [2]

c) Solve the differential equations: $e^{x-y}dx + e^{y-x}dy = 0$. [2]

[Ans: $e^{2x}+e^{2y}=C$ **]**

4. a) Find the regression Coefficients b_{yx} and b_{yx} for the data given below $\sum X=24;\sum Y=12;\sum X^2=374;\sum Y^2=97;$ $\sum XY=157;n=7$ [2]

[Ans: $b_{yx}=\frac{811}{2042};\ b_{xy}=\frac{811}{535}$ **]**

b) If the mean and variance of a binomial distribution are 9 and 6 respectively. Find the distribution. [2]

[Ans: $\left(\frac{2}{3}+\frac{1}{3}\right)^{27}$ **]**

c) What is the probability of getting a black or ace from a pack of 52 cards. [2]

[Ans: $\frac{7}{13}$ **]**

5. a) In how many ways can the letters of the word "CALCULUS" be arranged so that the two C's do not come together. [4]

[Ans: 3780**]**

b) Let $G=Q-\{-1\}$ the set of all rational numbers without –1. Suppose an operation * defined on G is given by $a*b=a+b+ab$. Show that the system is a group. [4]

6. a) Prove that, by using vector method. [4]

$$Cos(A-B)=CosACosB+SinASinB$$

OR

Prove, in any triangle by vector method.

$$\frac{SinA}{a}=\frac{SinB}{b}=\frac{SinC}{c}$$

b) Find the equation of the plane through the point (2, 1, 4) and perpendicular to each of the planes $9x-7y+6z+48=0$ and $x+y+z=0$. [4]

[Ans: $13x+3y-16z+35=0$ **]**

OR

Fin the direction cosines of two lines which satisfy the equations $l+m+n=0$ and $l^2+m^2+n^2=0$

[Ans: $0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ **]**

7. a) Evaluate: $\int \sqrt{\frac{1+x}{1-x}}\, dx$ [4]

[Ans: $Sin^{-1}x - \sqrt{1-x^2} + C$ **]**

b) Solve: $\frac{dy}{dx} + y\tan x = y^3 \sec x$ [4]

[Ans: $(C - 2\sin x)y^2 = \cos^2 x$ **]**

OR

$$xy\frac{dy}{dx} - y^2 = x^2$$

[Ans: $y^2 = x^2(\log x^2 + C)$**]**

8. a) Calculate the correlation coefficient from the following data: [4]

X:	20	22	15	25	23	16	18
Y:	12	18	10	20	16	15	14

[Ans: $r = 0.80$**]**

b) The probability of a bomb hitting a target is $\frac{1}{5}$. Two bombs are enough to destroy a bridge. If six bombs are aimed at the bridge, find the probability that the bridge is destroyed. [4]

9. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that $C_0C_2 + C_1C_3 + C_2C_4 + \ldots + C_{n-2}C_n = \frac{2n!}{(n-2)!(n+2)!}$. Also find the 7th term in the expansion of $\left(2x + \frac{1}{y}\right)^{10}$. [6]

[Ans: $\frac{3360x^4}{y^6}$ **]**

OR

Sum to infinity the given series $1 + \frac{1+2}{2!} + \frac{1+2+2^2}{3!} + \ldots.$

Also prove that: $\frac{1}{2.3} + \frac{1}{4.5} + \frac{1}{6.7} + \ldots = 1 - \log_e 2$

[Ans: $e^2 - e$ **]**

10. Derive the equation of the tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) on it. Find the equation of the tangent at (1, 1) to the parabola $y^2 = x$? [6]

[Ans: $yy_1 = 2a(x + x_1);\ 2y = (x+1)$**]**

11. State Lagrange's mean value theorem. Interpret it geometrically. Verify Lagrange's mean value theorem for the given function $f(x) = \log x$ on $[1, e]$ [6]

Group 'C'

16. a) Determine graphically the solution set of the following system of inequalities $x - y \le 5;\ \ 2x + y \ge 3$ [2]

b) Find the inverse of the given matrix by Gauss Jordan method.

$$A = \begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix}$$ [2]

[Ans: $A^{-1} = \begin{bmatrix} 1 & 0.333 \\ 1 & 0.667 \end{bmatrix}$ **]**

c) Given a tabulated values of the velocity of an object.

Time (s)	0.0	1.0	2.0	3.0
Velocity (m/s)	0	10	15	20

obtain an estimate of the distance travelled in the interval [0, 3]. [2]

[Ans: 35**]**

17. a) Use the Gauss-Seidel method to solve the systems $4x - y + z = 8;\ 2x + 5y + 2z = 3;\ x + 2y + 4z = 11$ [4]

[Ans: $x = 1, y = -1, z = 3$**]**

b) The parking area in a garage is 600 sq. m. The average area required for a car is 6 sq.m. and for a bus 30 sq.m. How many of each should the garage arrange to park for maximum income of a car pays Rs. 25 and a bus pays Rs. 75 and not more than 60 vehicles can be taken owing to the time required to handle them. [4]

[Ans: Maximum income = 2000 at (50, 10)**]**

OR

Use the simplex method to find the optimal solutions of the given L.P. problems Max $z = 9x + y$ Subject to

$$2x + y \le 8;\ 4x + 3y \le 18;\ \ x \ge 0;\ y \ge 0$$

[Ans: Max. Z = 36 at (4, 0)**]**

18. Apply the method of successive bisection to find the root of the equation $x^3 - 4x - 1 = 0$ lying between 1 and 2 correct to two places of decimal by successive bisection method. [6]

[Ans: $1.859375 \approx 1.86$**]**

OR

Find Newton's method to find the positive root of $Sinx + x - 1 = 0$ in (0, 1).

[Ans: $0.51097 \approx 0.511$**]**

19. Determine using a) Trapezoidal rule b) Simpson's rule, the following integrals. Estimate the error in each case $\int_1^3 \frac{dx}{x^2}$, $n = 4$. Estimate the error in using the approximation $n = 4$. [6]

[Ans: 0.705; 0.67145; Error: 0.0383 0.00475**]**

SET 3

Group 'A'

1. a) A bag contains 5 red balls. How many ways can select at most 3 balls? [2]

[Ans:26 ways**]**

b) Expand upto four term: $\frac{1}{\sqrt{1+2x}}$ [2]

[Ans: $1 - x + \frac{3}{2}x^2 - \frac{5}{2}x^3 + \ldots$**]**

c) Let $G = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$ and the operation defined on *G* be of addition. Find the identity and the inverse elements of 1 and 2. [2]
[Ans:–1 and –2 are the inverse of 1 & 2 respectively. And 0 is the identity of 1 & 2**]**

2. a) Deduce the equation of the ellipse in the standard position with the given data. A focus at (0, –5) and eccentricity 1/3. [2]

[Ans: $225x^2 + 200y^2 = 45000$**]**

b) Prove that $l^2 + m^2 + n^2 = 1$ where *l, m, n* are direction cosines of a line. [2]

c) Find a vector perpendicular to the plane of two vectors $3\vec{i} + \vec{j} + 2\vec{k}$ and $\vec{i} - 3\vec{j} + 5\vec{k}$. [2]

[Ans: $11\vec{i} - 13\vec{j} - 10\vec{k}$ **]**

3. a) Find the points on the curve where the tangents are parallel to the *x*-axis. $y = x^3 - 3x^2 + 1$ [2]

[Ans: $(0,1), (2,-3)$**]**

b) Evaluate: $\int \frac{dx}{(4x+3)\sqrt{x+3}}$ [2]

[Ans: $\frac{1}{6}\log\left(\frac{2\sqrt{x+3}-3}{2\sqrt{x+3}+3}\right)+C$**]**

c) Solve: $(1+x^2)dy=(1+y^2)dx;\ y(0)=3$ [2]

[Ans: $y-x=3(1-xy)$**]**

4. a) For a group of 10 items $\sum x=452, \sum x^2=24270$ and mode = 43.7. Find the Pearson's coefficients of skewness. [2]

[Ans: S_k = 0.0765**]**

b) Find the mean deviation from mean of the numbers
31, 35, 29, 63, 55, 72, 37 [2]

[Ans: 14.86**]**

c) Given $P(A)=0.4,\ P(AUB)=0.56,\ P(B)=0.3$. Are A and B independent? [2]

[Ans: No**]**

5. a) How many different permutations can be made with letters of the word RANDOM under the following conditions
   i) if permutations begin with D
   ii) if permutations end with N
   iii) if permutations begin with A and end with O
   iv) if vowels are never separated [4]

[Ans: i) 120 ii) 120 iii) 24 iv) 48**]**

b) If *a* and *b* are the elements of a group (G, 0), then *a*0*x* = *b* and *x*0*a* = *b* have unique solution in (G, 0). [4]

6. a) Show that the angle between the tangents to the parabolas $y^2=4x$ and $x^2=4y$ at their points of intersection other than the origin is $\tan^{-1}\left(\frac{3}{4}\right)$ [4]

OR

A double ordinate of the parabola $y^2=2ax$ is of length 4*a*. Prove that the lines joining the vertex to its ends are at right angles.

b) Define linear combination of vectors. Prove that the vectors $-\vec{a}+4\vec{b}+3\vec{c}$, $\overrightarrow{2a}-3\vec{b}-5\vec{c}$ and $2\vec{a}+7\vec{b}-3\vec{c}$ are coplanar, where $\vec{a}, \vec{b}, \vec{c}$ are any vectors. [4]

OR

Show that the vectors $5\vec{i}+6\vec{j}+7\vec{k}$, $7\vec{i}-8\vec{j}+9\vec{k}$ and $3\vec{i}+20\vec{j}+5\vec{k}$ are linearly dependent.

7. a) Evaluate: $\int \frac{x^2 dx}{(x^2+4)(x^2+9)}$ [4]

[Ans: $-\frac{2}{5}\tan^{-1}\frac{x}{2}+\frac{3}{5}\tan^{-1}\frac{x}{3}+C$ **]**

OR

Evaluate: $\int \sqrt{x^2-a^2}\,dx$

[Ans: $\frac{x}{2}\sqrt{x^2-a^2}-\frac{1}{2}a^2\log\left(x+\sqrt{x^2-a^2}\right)+C$ **]**

b) Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}\,dx$ [4]

[Ans: $y+\sqrt{x^2+y^2}=Cx^2$ **]**

8. a) Find the line of regression of Y on X for the following data: [4]

X	10	9	8	7	6	4	3
Y	8	12	7	10	8	9	6

[Ans: 3Y = 19+X**]**

b) A class consists of 60 boys and 40 girls. If two students are chosen at random, what is the probability that
i) Both are boys
ii) Both are girls
iii) One boy and one girl? [4]

[Ans: i) $\frac{59}{165}$ ii) $\frac{26}{165}$ iii) $\frac{16}{33}$ **]**

9. Define direction cosines of a line. Prove that the lines whose direction cosines are given by the relation $al+bm+cn=0$ and $fmn+gnl+hlm$ are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$. [6]

OR

Define plane. A variable plane is at a constant distance 3*p* from the origin and meets the axes in the points A, B, C. Prove that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{p^2}$.

10. If the r^{th} term in the expansion of $(1+x)^{20}$ has its coefficient equal to that of $(r+4)^{th}$ term, find *r*. Also expand $\sqrt{1-x}$ upto four terms. [6]

[Ans: $r=9;\ 1-\frac{1}{2}x-\frac{1}{8}x^2+\frac{1}{16}x^3+\ldots$ **]**

11. What do you mean by continuous function of $f(x)$ at $x = a$? Given

$$f(x)=\begin{cases} 2x+3 & \text{for} \quad x<3 \\ 3x & \text{for} \quad x\geq 3 \end{cases}$$

Show that $f(x)$ is continuous at x = 3. If $f(x)$ differentiable at x = 3? How? [6]

[Ans: No**]**

Group 'C'

16. a) Find the vertices of the feasible region under constants

$3x + 2y \leq 48$, $x + y \leq 20$, $x, y \geq 0$

b) Convert the hexadecimal numeral AFB2 to binary form. [2]

[Ans: 1010111110110010_2**]**

c) Define well conditioned and ill-conditioned of a system of equations.

17. a) Use Newton-Raphson's method to approximate $\sqrt[3]{2}$ with an error less than 0.00001. [4]

[Ans: $\sqrt[3]{2}=1.25992$; Error = 0.0000095**]**

OR

Determine the number of iterations required by bisection method necessary to solve $f(x)=x^3+4x^2-10=0$ with accuracy 10^{-3} using a $a_1=1$ and $b_1=2$.

[Ans: Iterations = 10**]**

b) Solve the given system of linear equation by using inverse matrix method. $3x+y+z=5;\ x-2y+3z=2;\ 2x-5y-z=-4$ [4]

[Ans: (1, 1, 1)**]**

18. Minimize $W=2x+y$

Subject to

$5x+y\geq 9;\ 2x+2y\geq 10;\quad x\geq 0;\quad y\geq 0$ [6]

[Ans: Maximum value = 6 at (1, 4)**]**

[Ans: $0.51097\approx 0.511$**]**

19. Evaluate an approximate area between the curve $y=(2x+1)^2,\ x=1,\ x=3$ and x-axis taking 4 intervals by Simpson's rule. Also compare it with exact value. [6]

[Ans: 52.66 sq. unit; Error: = 0.02%**]**

OR

Estimate the integrals $\int_0^3 x\,dx$ taking n = 6 sub intervals by Trapezoidal and Simpson's rule and compare the result with exact value.

[Ans: 4.5 sq. unit; Error = 0**]**

SET 4

Group 'A'

1. a) In how many different ways can a garland of 9 flowers be made? [2]

[Ans: 20160**]**

b) Prove that:

$$\frac{1}{n+1}+\frac{1}{2(n+1)^2}+\frac{1}{3(n+1)^3}+\ldots=\frac{1}{n}-\frac{1}{2n^2}+\frac{1}{3n^3}\ldots \quad [2]$$

c) If a,b,c are the element of a groups $(G, 0)$ and $a0b = a0c$, then $b = c$. [2]

2. a) Prove that the line $ty = x + at^2$ touches the parabola $y^2 = 4ax$ and find its point of contact. [2]

b) Find the length of the perpendicular from the point (2, 5, 7) to the plane $6x + 6y + 3z = 11$. [2]

[Ans: $5\frac{7}{9}$ units**]**

c) If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is angle between them, show that $\frac{1}{2}\left|\vec{a}-\vec{b}\right|=\sin\frac{\theta}{2}$. [2]

3. a) If $\vec{a}=\vec{i}+2\vec{j}+3\vec{k}$ and $\vec{b}=2\vec{i}+3\vec{j}+4\vec{k}$ find the projection of $\vec{a}$ on $\vec{b}$. [2]

[Ans: $\frac{20}{\sqrt{29}}$ **]**

b) Find the derivative of $x^{\cosh^2\frac{x}{a}}$. [2]

[Ans: $x^{\cosh^2\frac{x}{a}}\left\{\frac{1}{x}\cosh^2\frac{x}{a}+\frac{1}{a}\log x.\sinh\frac{2x}{a}\right\}$**]**

c) Prove that: $\int \operatorname{cosec} x\,dx=\log\left|\tan\frac{x}{2}\right|+C$ [2]

4. a) Solve: $\frac{dy}{dx}=\frac{xy+y}{xy+x}$ [2]

[Ans: $\log\frac{y}{x}+y-x=C$ **]**

b) Find the correlation coefficient between the two variables from the given data $n=10,\ \sum X=18; \sum Y=25; \sum X^2=90;$ $\sum Y^2=120; \sum XY=65$ [2]

[Ans: $r_{xy}=0.35$ **]**

c) If $P(A)=\frac{3}{8}$, and $P(B)=\frac{1}{2}$, $P(A\cap B)=\frac{1}{4}$; find $P\left(\frac{A}{B}\right)$ and $P\left(\frac{B}{A}\right)$. [2]

[Ans: $\frac{1}{2}$ and $\frac{2}{3}$ **]**

5. a) In how many ways can a committee of 15 members formed out of 8 engineers, 5 doctors and 4 lawyers so that all the lawyers always have a representation. [4]

[Ans: 78**]**

b) Define a binary operation. Test the closure, associative and commutative properties for the given case, the operation defined by $m*n=m+n+1\ \ m,n\in z$ [4]

6. a) Find the sum of the infinite series. [4]

$$\frac{1.2}{1!}+\frac{2.3}{2!}+\frac{3.4}{3!}+\ldots$$

[Ans: $3e$ **]**

OR

Prove that the middle term in the expansion of $(1+x)^{2n}$ is $\frac{1.3.5..(2n-1)}{n!}2^n x^n$.

b) OB and OC are two straight lines and D is a point on BC such that BD : DC = m : n, show that $\overrightarrow{OD}=\frac{n.\overrightarrow{OB}+m.\overrightarrow{OC}}{m+n}$ [4]

7. a) Find, from first principles, the derivative of: $\log x^x$ [4]

[Ans: $1+\log x$ **]**

OR

Evaluate: $\lim\limits_{x\to\frac{\pi}{2}}\frac{\tan 5x}{\tan x}$

[Ans: $\frac{1}{5}$ **]**

b) Evaluate: $\int \frac{dx}{4+3\sinh x}$

[Ans: $\frac{1}{5}\log \frac{1+2\tanh\frac{x}{2}}{4-2\tanh\frac{x}{2}}+C$ **]**

8. a) Solve the differential equation: [4]

$$\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$$

[Ans: $y=\frac{1}{2}e^{\tan^{-1}x}+Ce^{-\tan^{-1}x}$ **]**

b) State and prove theorem of compound probability. [4]

OR

In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the probability *p* of a success in a single trial.

9. Define conic section. If a normal chord (a chord which is normal to the parabola) of a parabola $y^2 = 4ax$ subtends a right angle at the vertex, show that it is inclined at an angle $\tan^{-1}\sqrt{2}$ to the axis. [6]

OR

Define ellipse. Show that $16x^2+25y^2+64x+50y-311=0$ represents the equation of an ellipse. Find its vertices, eccentricity, foci and length of latus rectum.

[Ans: $(3,-1)$ and $(-7,-1)$; $e=\frac{3}{5}$; $(1,-1)$; $(-5,-1)$; $\frac{32}{5}$ **]**

10. Derive the angle between two lines whose d.c's are (l_1, m_1, n_1) and (l_2, m_2, n_2). Also derive the condition of perpendicularity and parallism.[6]

11. Find the mean, standard deviation and coefficient of variation of the following data: [6]

Wages in (Rs.)	20	25	28	35	36	40	47	51	58
No. of workers	5	4	8	10	5	7	6	3	2

[Ans: mean = 35.7; S.D. = 9.76; C.V.=27.34%**]**

Group 'C'

16. a) Convert to decimal 10110110_2 [2]

[Ans: 182**]**

b) Test the consistency of the given system of equations: [2]

$$-2x + y + 3z = 12;\ x + 2y + 5z = 4;\ 6x - 3y - 9z = 24$$

[Ans: inconsistant**]**

c) Evaluate the given integrals using Trapezoidal rule. Give your answer correct upto 3 places of decimals.

$$\int_0^1 x^2 dx,\ n = 2$$ [2]

[Ans: 0.375**]**

17. a) Apply successive bisection method to find the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to two places of decimal.[4]

[Ans: 1.86

OR

Maximize $Z = 6x_1 - 9x_2$ Subject to

$$2x_1 - 3x_2 \le 6;\ x_1 + x_2 \le 20;\ x_1 \ge 0;\ x_2 \ge 0$$

By using simplex method.

[Ans: Z = 18 at (3, 0)**]**

b) Compute two approximate values for $\int_1^2 x^{-2} dx$ using $h = \frac{1}{2}$ and $h = \frac{1}{4}$ with the composite Trapezoid rule. [4]

[Ans: 0.509)**]**

18. Solve the given system of equations by Gauss-Seidel method.

$$5x + 3y - z = 15;\ 2x - 4y + z = -4;\ x - y - 3z = -13$$ [6]

[Ans: $x = 2,\ y = 3,\ z = 4$**]**

OR

Solve the system of equations by using Gauss Jordan method

$$2x - y + z = -2;\ x + y - 2z = -9;\ x + 2y + z = 9$$

[Ans: $x = 2.005;\ y = 2.9981;\ z = 5.0011\ \therefore (x, y, z) = (-2, 3, 5)$**]**

19. Find by Newton's method, the root of $e^x - 4x$ which approximately 2, correct to three places of decimals. [6]

[Ans: 2.1532**]**

SET 5

Group 'A'

1. a) A man has 15 friends of whom 10 are relatives. In how many ways can he invite 8 guests such that 5 of them may be relatives? [2]

[Ans: 2520**]**

b) Prove that e lies between 2 and 3. [2]

c) If a binary operation * on Q the set of rational number is defined by $a * b = a + b - ab$ for every $a, b \in Q$. Show that * is commutative and associative. [2]

2. a) Determine the equation to the hyperbola in standard form with a vertex at (0, 8) and passing through $\left(4, 8\sqrt{2}\right)$. [2]

[Ans: $\frac{y^2}{16} - \frac{x^2}{16} = 1$**]**

b) Find the angle between two lines whose direction ratios are 2, 3, 4 and 1, –2, 1. [2]

[Ans: $\frac{\pi}{2}$ **]**

c) Find the distance between the parallel planes $2x - 2y + z + 1 = 0$ and $4x - 4y + 2z + 3 = 0$ [2]

[Ans: $\frac{1}{6}$ **]**

3. a) Find the value of *m* if the vectors $3\vec{i} - \vec{j} - 2\vec{k}$ and $2\vec{i} - 2\vec{j} + m\vec{k}$ are orthogonal. [2]

[Ans: *m* = 4**]**

b) Show that $f(x) = |x - 1|$ is not differentiable at *x* = 1. [2]

c) Evaluate: $\int \frac{a^x dx}{\sqrt{1 - a^{2x}}}$ [2]

[Ans: $\frac{1}{\log a} Sin^{-1} a^x + C$ **]**

4. a) Solve: $y(1 + xy)dx - xdy = 0$ [2]

[Ans: $\frac{x}{y} + \frac{1}{2}x^2 = C$ **]**

b) Following are the information about the marks of two student A and B.

	A	B
Average Mark	84	92
Variance of Marks	16	25

Examine who has got the uniform mark. [2]

[Ans: A**]**

c) Find the probability of getting two heads twice in 5 tosses of two coins.[2]

[Ans: $\frac{315}{512}$**]**

5. a) How many words can be formed from the letters of the word "ENGLISH"? [4]

i) How many of these do not begin with E?

ii) How many of these begin with E and do not end with H?

[Ans: 5040, 4320, 600**]**

b) Prove that the set $G = \{1, w, w^2\}$. Where w is an imaginary cube root of unity is a group under the usual rules of multiplication. [4]

6. a) If $y = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ Show that $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$ [4]

OR

Prove that:

$$\frac{\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots}{\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots} = \frac{e-1}{e+1}$$

b) The projection of a line on the axes are 6, 2, 3. Find the length of the line and its direction cosines. [4]

[Ans: $\text{length} = 7;\ \text{d.c's} = \frac{6}{7}, \frac{2}{7}, \frac{3}{7}$ **]**

OR

Find the ratio in which the line joining the points (–3, 4, 8) and (5, –6, –4) is divided by the xy plane. Also, find the co-ordinates of the point of inter-section of the line with the plane.

[Ans: $2:1;\ \left(\frac{7}{3}, -\frac{8}{3}, 0\right)$ **]**

7. a) Find the derivative by using first principle of: $\tan^{-1} x$ [4]

[Ans: $\frac{1}{1+x^2}$ **]**

b) Evaluate: $\int \frac{dx}{(x+1)^2 (x-2)^3}$ [4]

[Ans: $-\frac{1}{243}\left\{\frac{1}{2}\left(\frac{x+1}{x-2}\right)^2 - 3\left(\frac{x+1}{x-2}\right) + 3\log\left|\frac{x+1}{x-2}\right| + \left(\frac{x-2}{x+1}\right)\right\} + C$ **]**

8. a) Marks of two students in six examinations out of total score 100 were as follows. [4]

Student A	40	60	70	80	50
Student B	60	30	80	70	60

Find which student may be considered to be more consistent. [4]

[Ans: Student A**]**

OR

For the frequency distribution given below, calculate appropriate coefficient of skewness.

Monthly income (in Rs.)	Below 100	100-150	150-200	200-250	250-300	300 and above
No. of workers	10	25	145	220	70	30

[Ans: S_k = –0.1022**]**

b) Suppose 3 cards are drawn from a well shuffled deck of 52 cards. What is the probability of getting
   i) all three spades
   ii) two aces [4]

[Ans: i) $\frac{11}{850}$ ii) $\frac{72}{5525}$**]**

9. Define linearly dependent and independent vectors. Show that the three points whose position vectors are $2\vec{i}-\vec{j}+\vec{k},\ \vec{i}-3\vec{j}-5\vec{k}$ and $3\vec{i}-5\vec{j}-4\vec{k}$ form the sides of right angled triangle. [6]

10. Solve the differential equation

$$x\frac{dy}{dx}+x\cos^2\frac{y}{x}=y$$

[6]

[Ans: $\tan\frac{y}{x}+\log|x|=c$**]**

OR

Solve the given differential equation $\frac{dy}{dx}$ = y tanx – 2 sinx

[Ans: $y\cos x=\frac{\cos 2x}{2}+c$**]**

11. Define focal chord and latus rectum of a parabola. Also prove that the latus rectum of a parabola bisects the angle between the tangent and normal at either extremity of the latus rectum. [6]

Group 'C'

16. a) Find the solution of the given equation by Gauss Seidel method after first iteration $3x-2y=8,\quad x+3y=-1$ [2]

[Ans: $8/3,\ -11/9$**]**

b) Convert the given binary numerals into hexadecimal form. 1010111010. [2]

[Ans: 2BA**]**

c) Find all basic feasible solutions of the given system of equations

$2x + y = 3$; $x - y + z = 2$. [2]

[Ans: $x = 1$, $z = 1$ (basic); $y = 0$; $y = 1$, $z = 3$ (basic); $x = 0$**]**

17. a) Test the ill-conditioned of following equations:

$211x - 125y = 47$

$405x - 240y = 90$ [4]

OR

Solve the given system of equation by using Gauss elimination method.

$x + 2y + 3z = 2;\ x + y - z = 1;\ 2x + 3y + 2z = 3$

[Ans: $x = k$, $y = 1 - 4k$ *and* $z = 5k$**]**

b) Write three methods for measuring error. Show that a root of the equation $x^3 - 3x - 5 = 0$ lies in the open internal (2, 3). Find this root approximately by Newton-Raphson's method using two iterations only approximate x_1, x_2 correct to two decimal. [6]

[Ans: Required root = 2.28 (Approx.)**]**

18. Maximize $P = 50x + 80y$ by using simplex method. Subject to: $x + 2y \le 32;\ 3x + 4y \le 84;\ x \ge 0;\ y \ge 0$ [6]

[Ans: Max P = 1480 at (20, 6)**]**

OR

Minimize $P = 12x + 10y$ Subject to:

$x + y \ge 6;\ 2x + y \ge 8, x \ge 0;\ y \ge 0$

[Ans: Optimal Solutions P = 64 at (2, 4)**]**

19. Calculate the value of $\int_0^1 \frac{1}{1+x^2}dx$ taking 5 sub intervals by Trapezoidal rule correct to four significant figures. Also estimate the error with its exact value. [4]

[Ans: 0.7837, Error = 0.002**]**