Mathematics - XI

SET - I

Group A **(5×3×2 =30)**

1. a. Find negation of the following statements:
   i) 10 is a composite number
   ii) 2+3=5 or 10 is multiple of 5

   b. If A={1, 2, 3}, find the relation on A satisfying the condition x+y<4. Is this relation a function? Give reason.

   c. Examine whether the function f(x) = $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ is even or odd. Also examine for its symmetricity.

2. a. Prove that: $\tan^{-1} a - \tan^{-1} c = \tan^{-1}\frac{a-b}{1+ab} + \tan^{-1}\frac{b-c}{1+bc}$

   b. Sum to infinity the series: $1-5a+9a^2-13a^3+\ldots\ldots\ldots$to ∞ $(-1<a<1)$

   c. If $A = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$, find the value of A^3.

2. a. Using Cramer's rule, solve the system of equations.
   $$3x + 4y = -2$$
   $$5x - 7y - 24 = 0$$

   b. Express $-1-\sqrt{3}\,i$ into polar form.

   c. If the equation $x^2 + 2(a + 2)x + 9a = 0$ has equal roots, find a.

3. a. If the equation $2x^2 + 7xy + 3y^2 - 4x - 7y + c = 0$ represent a pair of line, find the value of c.

   b. Find the equation of a circle with centre at (3, 4) and touching X-axis.

   c. Evaluate: $\lim\limits_{x \to 0} \frac{1-\cos x}{x^2}$

4. a. Find $\frac{dy}{dx}$ if x+y = sin(x+y).

   b. Evaluate: $\int \frac{1}{x}\sin(\log x)\,dx$

   c. A spherical ballon is inflated at the rate of 10 cubic cm/sec. At what

rate is the radius increasing when the radius is 10 cm?

Group B **(5×2×4=40)**

5. a. If A,B and C be any three non-empty sets, prove that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

Or

Solve the inequality $|2x+1|>3$. Represent the solution in a real line.

b. Draw the graph of $y=2^x$ indicating its different characteristics.

6. a. Solve: $\sin x+\cos x=\sqrt{2}$

Or

State sine law. In triangle ABC, prove that $\tan\frac{B-C}{2}=\frac{b-c}{b+c}\cot\frac{A}{2}$

b. Prove that $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & xz & xy \end{vmatrix}=(y-z)(z-x)(x-y)(yz+zx+xy)$

7. a. Solve the following system of linear equations by row equivalent matrix method **or** inverse matrix method.

$2x+6y=2,\quad 3x-z=-8,\quad 2x-y+z=-3$

b. If $x^2+px+q=0$ and $x^2+qx+p=0$ have a root in common, prove that $p=q$ or $p+q+1=0$.

8. a. If $x+y=2$ is the equation of the chord of the circle $x^2+y^2-2y=0$, find the equation of the circle so that this chord is a diameter

b. Evaluate $\lim\limits_{x\to\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)$

OR

A function f(x) is defined as follows:

$$f(x)=\begin{cases}\dfrac{x^2-x-6}{x^2-2x-3} & for\,x\neq 3\\[2ex] \dfrac{5}{3} & for\;x=3\end{cases}$$

Prove that f(x) is discontinuous at x=3. Can the function be modified so as to make it continuous at x=3?

9. a. Find from the definition, the derivative of $\sqrt{\tan x}$

b. Find area between the curves $y^2=x$ and $x^2=y$.

Group C **(5×6=30)**

10. Define bijective function with an example. Let R be the set of real numbers. Show that the function $f: R \to R$ define by $f(x) = 4x - 7$ is bijective and find the formula for f^{-1}.

11. The AM, GM and HM between two unequal positive numbers satisfy the relations
    i. AM × HM = $(GM)^2$ ii. AM > GM > HM

12. Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are right angle if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$.

OR

Find the angle between the lines $y = m_1 x + c_1$ and $y = m_2 x + c_2$. Also find the condition of parallelism and perpendicularity of the lines. Hence find the angle between the lines $x - \sqrt{3}y = 5$ and $\sqrt{3}x - y = 1$.

13. State De-Moivre's theorem. Using De-Moivre's theorem, find the square roots of $4 + 4\sqrt{3}i$.

14. List the criteria for the function y=f(x) to have local maxima and local minima at a point. Find the local maxima and minima of $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (-1, 2), also find the point of inflection.

OR

A ladder 5 meters long rests against a vertical wall. If its top slides down wards at the rate of 10 cm/s, find the rate at which the foot of the ladder is sliding when the foot of the ladder is 4 meters away from the wall.

SET - II

Group A **[5×3×2=30]**

1. a. Define a statement. If p is true, q is false and r is true, find the truth value of $(p \vee q) \wedge (\sim q)$.
   b. Find domain of the function $y = \sqrt{x-3}$.
   c. Test periodicity and symmetricity of the function y = tan 2x.
2. a. Solve: $\cos^{-1} x - \sin^{-1} x = 0$

b. Using principle of mathematical induction, prove that : $1 + 3 + 5 + \ldots + (2n-1) = n^2$.

c. If $\begin{pmatrix} -3 & -2 \\ 5 & 3 \end{pmatrix}$ and $\begin{pmatrix} x & y \\ -5 & -3 \end{pmatrix}$ are inverse matrices of each other, find x and y.

3. a. Using Inverse matrix method, solve
$2x + y = 8, \quad x - 2y = -1$

b. If $\alpha = \frac{-1 + \sqrt{-3}}{2}$ and $\beta = \frac{-1 - \sqrt{-3}}{2}$, prove that $\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$.

c. Prove that the roots of $x^2 - 3x + 2 = 0$ are rational.

4. a. Find the distance between two parallel lines $3x + 4y = 17$ and $6x + 8y + 1 = 0$

b. Find the equation of a circle concentric with $x^2 + y^2 - 4x + 6y + 7 = 0$ and touching Y-axis.

c. Evaluate: $\lim_{\theta \to \pi/4} \frac{\cos\theta - \sin\theta}{\theta - \pi/4}$

5. a. Find $\frac{dy}{dx}$ of $y = e^{\sin(\log x)}$

b. Evaluate: $\int \ln x^2 \, dx$

c. A stone thrown into a pond produces circular ripples which expands from the point of impact. If the radius of the ripple increases at the rate of 3.5 cm/sec, find how fast is the area growing when the radius is 15 cm?

Group B **[5×2×4 = 40]**

6. a. Define the complement of a set. State and prove De-Morgan's Laws.

OR

Define absolute value of a real number. For any two real numbers x and y prove that: $|x + y| \le |x| + |y|$

b. Sketch the graph of the function $y = \frac{1}{x-2}$ giving its different characteristics.

7. a. In any triangle ABC
If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{(a+b+c)}$ show that $\angle C = 60°$.

OR

If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $xy + yx + zx = 1$

b. If a, b, c are non-zero and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$
then show that, $abc = 1$.

8. a. Using row equivalent method **or** inverse matrix method, solve the system of linear equations:

$$\begin{aligned} x + 2y - z &= 8 \\ 2x + 3y + z &= 5 \\ 3x + y + 2z &= -1 \end{aligned}$$

b. Under what condition will quadratic equation $ax^2 + bx + c = 0$ has,

i. reciprocal roots

ii. roots equal in magnitude but opposite in sign

9. a. Obtain the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

b. Evaluate: $\lim\limits_{x \to \theta} \dfrac{x \sin\theta - \theta \sin x}{x - \theta}$

OR

Show that the function $f(x) = \begin{cases} x & 0 \le x < 1/2 \\ 1 & x = 1/2 \\ 1-x & 1/2 < x < 1 \end{cases}$

is discontinuous at $x = \dfrac{1}{2}$. Also, write how it could be made continuous?

10. a. Find from the definition, the derivative of $\sin^2 3x$.

b. Find the area enclosed by the curve $x^2-8x+15=0$ with x-axis.

Group C **[5×6 = 30]**

11. Define one to one function and onto function with suitable example. Determine whether the function $f : R \to R$ given by $f(x) = x^2 - 1$ is one to one or onto or both or neither.

12. If G_1 and G_2 are two geometric means between b and c and a is their arithmetic mean, then show that $G_1^3 + G_2^3 = 2abc$. Also prove that A.M., G.M. and H.M. between two unequal positive quantities satisfy A.M. > G.M. > H.M.

13. The origin is a corner of the square and two of it's sides are $y + 2x = 0$ and $y + 2x = 3$. Find the equations of other two sides.

OR

Prove that the product of the perpendicular from (α, β) to the lines given by $ax^2 + 2hxy + by^2 = 0$ is $\dfrac{a\alpha^2 + 2h\alpha\beta + b\beta^2}{\sqrt{(a-b)^2 + 4h^2}}$. Also find the acute angle of bisector between the lines $4x + 3y - 7 = 0$ and $24x + 7y - 31 = 0$.

14. Define absolute value of complex number. If Z and W are two complex numbers, prove that $|Z+W| \le |Z|+|W|$.

15. For a function, write the conditions so that its graph has concave upward and concave downward. Find the interval where given function

$f(x) = x^4 - 8x^3 + 18x - 24$ is concave upward and concave downward. Also find the point of inflection.

OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate 8 cm/sec. and that of the outer circle at the rate of 5 cm/sec. At a certain instant the radii of the inner and outer circles are respectively 24 cm and 30 cm. At what rate does the area between the two circles changes.

SET – III

Group A **[5×3×2=30]**

1. a. Write inverse and converse of the statement "If two triangles are similar then their corresponding sides are proportional."

   b. Rewrite $-1 \le x \le 5$ using absolute value sign.

   c. Examine whether the function $f(x) = 3x^2 + \cos x$ is even or odd. Also test for symmetricity.

2. a. Solve: $\tan\theta + \cot\theta = 2$ $(0 \le \theta \le \pi)$

   b. Using principle of mathematical induction, prove that "$x^n - y^n$ is divisible by $x - y$."

   c. If $A = \begin{pmatrix} 2 & -3 \\ 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix}$, find $(AB)^T$.

3. a. Using inverse matrix method, solve
   $x + 2y = 5$; $3x - y = 1$

   b. Express $\dfrac{i}{1-i}$ in the polar form.

   c. Find the value of k so that the equation $(3k + 1)x^2 + 2(k + 1)x + k = 0$ has reciprocal roots.

4. a. Find the equation of a straight line passing through origin and perpendicular to the line $3x - 5y = 7$.

   b. Find equation of a circle whose two of the diameters are $2x + y = 10$ and $x - y + 1 = 0$ and passing through origin.

   c. Evaluate: $\lim\limits_{x\to\infty} (\sqrt{x} - \sqrt{x-a})$

5. a. Find $\dfrac{dy}{dx}$ if $x^3 + y^3 - 3xy = 0$.

   b. Evaluate: $\displaystyle\int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$

c. Find the intervals in which $f(x) = x^2 - 2x + 10$ is increasing or decreasing.

Group B **[5×2×4 = 40]**

6. a. If A, B and C be three non-empty sets, prove that
$A - (B \cup C) = (A - B) \cap (A - C)$.

OR

Define absolute value of real number. Rewrite the given relation without using absolute value sign $|2x - 1| \leq 5$. Also, draw the graph of the inequality.

b. Sketch the graph of $f(x) = (x - 4)^2 - 8$ indicating its characteristics.

7. a. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, show that $x + y + z = xyz$.

OR

Solve the triangle if $a = \sqrt{6}$, $b = 2$ and $c = \sqrt{3} - 1$.

b. Prove that:

$$\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2)$$

8. a. Applying row equivalent matrix method **or** inverse matrix method, solve the following system of linear equations:

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 3z &= 4 \\ x + 3y + 7z &= 13 \end{aligned}$$

b. Find the condition for two given quadratic equations $p_1x^2 + q_1x + r_1 = 0$ and $p_2x^2 + q_2x + r_2 = 0$ may have one root common.

9. a. Find the equation of the line through $(1, -1)$ which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$.

b. Evaluate: $\lim\limits_{x \to 0} \dfrac{(a + x)\sec(a + x) - a\sec a}{x}$

OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x + 3 & \text{for } x < 1 \\ 3 & \text{for } x = 1 \\ 6x - 1 & \text{for } x > 1 \end{cases}$$

Is the function continuous at $x = 1$? If not how can you make it continuous?

10. a. Find the derivative of $\sqrt{\sin 2x}$ by the definition.

b. Using method of integration, find the area of the circle $x^2 + y^2 = r^2$

Group C [5×6 = 30]

11. Let a function f: A $\rightarrow$ B be defined by $f(x) = \frac{x+1}{2x-1}$ with A = {–1, 0, 1, 2, 3, 4} and B = $\{-1, 0, \frac{4}{5}, \frac{5}{7}, 1, 2, 3\}$. Find the range of f. Is the function *f* one-one and onto both? If not, how can the function be made one-one and onto both?

12. Define sequence and series. Find the n^{th} term and then sum of first n term of the series 6 + 13 + 24 + 29 + ……

13. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to the line $x \cos\alpha + y \sin\alpha = p$. Also find, the distance between the parallel lines $3x + 5y = 11$ and $3x + 5y = -23$

OR

Find the condition that the general equation of second degree may represent a pair of lines. For what value of k will the equation $x^2 - kxy + 4y^2 + x + 2y - 2 = 0$ represent a pair of straight lines.

14. Find cube roots of unity. Also, establish the properties of cube roots of unity.

15. Write the conditions for a function to have local maximum and local minimum value of a function. Find the maxima or minima for the given function $4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection.

OR

Water is running into a conical reservoir, 10 cm deep and 5 cm in radius at the rate of 1.5 cm^3 /min.

i. At what rate is the water level rising when the water is 4 cm deep.

ii. At what rate is the area of water surface of the reservoir increasing when the water is 6 cm deep.

SET – IV

Group A [5×3×2=30]

1. a. If p and q are any two statements, prove that $\sim(p\square q) \equiv (\sim p)\wedge(\sim q)$.

   b. Check whether the function $f:[-2,3] \longrightarrow R$ given by $f(x)=x^2$ is one to one, onto or both.

   c. What are the odd and even functions? Determine whether the function $f(x) = 2^x + 2^{-x}$ is even, odd or neither.

2. a. Solve: tanx +tan2x= tan3x.

   b. If H is the harmonic mean between a and b, prove that $(H-2a)(H-2b)=H^2$

   c. For the given, matrices $A=\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$. Show that $(A+B)^T= A^T+B^T$.

3. a. Can you solve the system 3x+4y=10, 6x+8y=24 by the inverse matrix method, if not why?

b. Prove that $\dfrac{a+bw+cw^2}{b+cw+aw^2}=\omega$ where ω,ω^2 are complex cube roots of unity.

c. If one root of the equation $ax^2+bx+c=0$ be twice the other show that $2b^2 = 9ac$.

4. a. Find the angle between the pairs of line represented by $7x^2+8xy+y^2=0$

b. Find the equation of the circle which has the points (0,-1) and (2,3) as ends of a diameter.

c. Evaluate $\lim\limits_{x\to 0}\dfrac{1-\cos qx}{1-\cos px}$

5. a. Find the derivative of $\dfrac{1}{\sqrt[3]{3x^2+x-5}}$

b. Show that the function $y=x^3-3x^2+6x+3$ has neither maximum nor minimum value.

c. Evaluate $\int \sec x\, dx$

Group B **[5×2×4 = 40]**

6. a. What is contradiction? Prepare the truth table for the compound statement $(p\wedge \sim q)\wedge(\sim p\vee q)$.

Or

Write the axiom of addition which are satisfied by the field of real numbers. Also solve the inequality; $x(x-2)(x+3)\le 0$.

b. Sketch the graph of the function $y=-x^2-2x+5$ indicating its different characteristics.

7. a. In any ΔABC if $\dfrac{\sin A+\cos A}{\cos B}=\sqrt{2}$ prove that $\angle C=135^0$.

Or

If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\dfrac{\pi}{2}$, prove that $xy+yz+zx=1$

b. Write the any two properties of determinant? Hence prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$

8. a. Solve; 3x+ 4y +5z =18; 2x - y +8z=13; 5x - 2y+7z=20 by using Row – equivalent matrix method or inverse matrix method.

b. Prove that the quadratic equation $ax^2+bx + c=0$ $(a \neq 0)$ cannot have more than two roots.

9. a. Find the equations of tangents drawn from the point (4,-2) to the circle $x^2 + y^2 = 10$. Also show that they are at right angle.

b. Evaluate: $\lim_{y \to x} \dfrac{y \sec y - x \sec x}{y - x}$

OR

When a function y=f(x) has a removable discontinuity at a given point x=a? A function f(x) is defined as,

$$f(x) = \begin{cases} 5x^2 + 3 & \text{for } x > 1 \\ 9 & \text{for } x = 1 \\ 6x + 2 & \text{for } x < 1 \end{cases}$$

Prove that f(x) has a removable discontinuity at x=1.

10. a. Find from the first principle the derivative of $\frac{1}{\sqrt{4-3x}}$

b. Find the area of ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ by using integration.

Group C **[5×6 = 30]**

11. Define domain and range of a function. Find the domain and range of the function $y = \sqrt{6 - x - x^2}$

12. Using the principle of mathematical induction prove that $1^2+2^2+3^2+\ldots+ n^2 = \dfrac{n(n+1)(2n+1)}{6}$.

13. State De Moivre's theorem. Use it find the fourth roots of i .

14. Derive the formula for the length of the perpendicular form a point (x_1,y_1) to a line ax+by+c = 0.Also find the distance between the parallel lines 5x+12y+8=0 and 10x+24y-3=0

Or

Prove that the bisectors of the angle between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is given by $\dfrac{x^2 - y^2}{xy} = \dfrac{a-b}{h}$.

15. A gardener having 120 meter of fencing wishes to enclose a rectangular plot of land and also erect a fence across a land parallel to two sides. Find the maximum area he can enclose.

Or

The volume of a spherical balloon is increasing at the rate of 25 cubic cm/sec. Find the rate of change of its surface at the instant when its radius is 5 cm.

SET – V

Group A **[5×3×2=30]**

1. a. Write the truth table of the statement $p \wedge (p \Rightarrow q) \Rightarrow q$. Draw a conclusion from a table.

   b. Examine whether the function $y = \cos\frac{x}{3}$ is odd or even. Also, discuss for its symmetricity and periodicity.

2. a. Prove that: $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$

   b. Prove by the principle of mathematical induction: 2+4+6+… …+2n = n(n+1)

   c. If A= $\begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}$, prove that $A+A^T$ is a symmetric matrix.

3. a. Using Cramer's rule, solve: x-2y = 0; 3x + 7y = 5 .

   b. If w be a complex cube root of unity, find the value of :$(1-w+w^2)^4(1+w-w^2)^4$.

   c. For what value of p will the equation $5x^2 - p x + 45 = 0$ has equal roots.

4. a. What are the points on x-axis whose perpendicular distance from the straight line $\frac{x}{a}+\frac{y}{b}=1$ is 'a'?

   b. Find the equation of the circle concentric with the circle $x^2+y^2 - 8x +12y + 15= 0$ and passing through at the point (5, 4).

   c. Prove that: $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$

5. a. Find $\frac{dy}{dx}$ when $y = \tan^{-1}\left(\frac{\sin 2x}{1+\cos 2x}\right)$.

b. The side of a square sheet is increasing at the rate of 5 cm/min. At what rate is the area increasing when the side is 12 cm. long?

c. Evaluate: $\int \cot x\,(\log \sin x)^{3}\,dx$

Group B **[5×2×4 = 40]**

6. a. Define symmetric difference of two sets. Prove that $A-(B\cup C)=(A-B)\cap(A-C)$

Or

Define the absolute value of real number. For any positive real number a, prove that $|x|\le a \Rightarrow -a\le x\le a$

b. Sketch the graph of the function $y=\sin x$ indicating its different characteristics.

7. a. Solve : $2\sin^2 x - 3\sin x\cos x + 3\cos^2 x = 1$

Or

State and prove that sine law of trigonometry.

b. Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

8. a. Using row equivalent method or inverse matrix method, solve: $3y+2z = 1$; $x+2y = -5$; $2x-z = -3$

b. If roots of the quadratic equation $x^2+px+q=0$ are in the same ratio as those of the equation $x^2+ax+b=0$, prove that $p^2b=a^2q$.

9. a. Find the equation of the line through the point (1,-1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2+y^2-6x+4y-3=0$.

b. Evaluate: $\lim\limits_{x\to 2} \dfrac{x-\sqrt{8-x^2}}{\sqrt{x^2+12}-4}$

OR

Define continuity of a function at a point. A function f(x) is defined as follows:

$$f(x)=\begin{cases} 2ax+3 & for\,x<1 \\ 1-ax^2 & for\,x\ge 1 \end{cases}$$

If f(x) is continuous at x=1, find the value of a.

10. a. Find from the first principle the derivative of $\sqrt{\sec x}$

b. Find the area of the region between the curves $y^2 = 16x$ and the line $y = 2x$.

Group C **[5×6 = 30]**

11. What is function? Find the domain and range of the function $y=\sqrt{36-x^2}$.
12. Find the n^{th} term and the sum of the n-term of the series: $1^2.2+2^2.3+3^2.4+\ldots$..
13. Define the conjugate of a complex number. Mention its geometrical meaning in Argand diagram. Also find the square roots of $\dfrac{8-6i}{1+i}$.
14. Find the equation of the bisectors of the angle between the straight lines 3x+4y+2=0 and 5x-12y-6=0. Verify that bisectors are perpendicular to each other. Also, identify the acute angle bisector.

    Or

    What is the homogenous equation of degree two? Prove that the two lines represented by $(x^2+y^2)\sin^2\alpha=(x\cos\theta-y\sin\theta)^2$ include an angle 2α.
15. A door is in the form of a rectangle surmounted by a semi-circle. If total perimeter is 9m, find the radius of semi-circle for the greatest door area.

    Or

    Water is running into a conical reservoir, 10cm deep and 5cm in radius at the rate of 1.5 cm^3/min. At what rate is the level rising when the water is 4 cm deep?