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DASHAIN VACATION ASSIGNMENT

Basic Mathematics – XI (Mgmt.)

Set, Real number system and Logic:

Short Questions:

1. Define power set. Write the power set of the set {a, b, c, d}.
2. If $A = \{x : x = 2n - 1, n \le 6, n \in \mathbb{N}\}$, $B = \{x : x = 3n + 1, n \le 3, n \in N\}$ then find (a) $A \cap B$ (b) $A \cup B$ (c) $A \Delta B$
3. Define the following with suitable examples.
   a. Intersection of two sets
   b. Union of two sets
   c. Complement of a set
   d. Symmetric difference of two sets.
   Also, show the examples in venn-diagram.
4. Rewrite the following without using absolute value sign $|2x + 3| \le 2$.
5. Rewrite $-11 < x < 6$ using absolute value sign.
6. Define absolute value of a real number. Prove that
   $|x + y| \le |x| + |y|$

Long Questions:

1. If $U = \{x : 1 \le x \le 10, x \in N\}$, $A = \{x : x < 8, x \in N\}$, $B = \{x : x \ge 6, x \in N\}$, verify that
   a. $A - B = \overline{B} - \overline{A}$
   b. $\overline{A \cap B} = \overline{A} \cup \overline{B}$
2. Let $A = [-2, 4]$ and $B = (2, 5]$; Compute
   a. $A \cup B$ b. $A \cap B$
   c. $A - B$ d. $B - A$
3. If A, B and C are subsets of the universal set U, prove that:
   a. $A - (B \cap C) = (A - B) \cup (A - C)$.
   b. $A \cup (B - C) = (A \cup B) - (C - A)$
   c. $A \Delta B = (A \cup B) - (A \cap C)$
   d. $\overline{A \cup B} = \overline{A} \cap \overline{B}$
   e. $\overline{A \cap B} = \overline{A} \cup \overline{B}$
   f. $A \cap (B - C) = (A \cap B) - (A \cap C)$
4. Solve and draw the graph of followings:
   a. $|2x - 7| \le 4$
   b. $|x - 1| \ge 1$.

Matrices:

Short questions: (Matrix)

1. Construct a 3 × 3 matrix A whose elements a_{ij} are given by $a_{ij} = 3j - 2j$.
2. If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that $(A - 2I)(A - 3I) = 0$ where I and O are unit matrix and zero matrix of order 2.
3. Define the following with examples:
   a. Transpose of a matrix
   b. Singular and non-singular matrix
   c. Inverse of a matrix
   d. Symmetric and skew-symmetric matrix
4. If $A = \begin{pmatrix} 2 & 0 & -6 \\ 5 & 1 & 2 \\ 7 & -3 & 0 \end{pmatrix}$, find A^T.
   i. Show that sum of the given matrix and its transpose is the symmetric matrix.
   ii. Show that difference of the given matrix and its transpose is the skew-symmetric matrix.
5. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, show that $A^{-1} = \frac{1}{19} . A$
6. State the conditions under which two matrices can be added and multiplied. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$ are two matrices, verify that $(AB)' = B'A'$.

Long questions: (Determinant)

1. Prove that:

i. $\begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = x^2\ (x+a+b+c)$

ii. $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

iii. $\begin{vmatrix} 1 & bc & b+c \\ 1 & ca & c+a \\ 1 & ab & a+b \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

iv. $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+1\right)$

v. $\begin{vmatrix} x^2+1 & xy & xz \\ xy & y^2+1 & yz \\ xz & yz & z^2+1 \end{vmatrix} = 1+x^2+y^2+z^2$

vi. $\begin{vmatrix} a & b & ax+by \\ b & c & bx+cy \\ ax+by & bx+cy & 0 \end{vmatrix} = (b^2-ac)\ (ax^2+2bxy+cy^2)$

2. If $xyz + 1 = 0$, show that

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$

System of linear equation:

Short Questions:

1. Solve the following equations using Cramer's rule:

i. $\frac{2x}{3} + y = 16$, $x + \frac{y}{4} = 14$

ii. $\frac{3}{x} + \frac{2}{y} = \frac{19}{20}$, $\frac{4}{x} + \frac{10}{y} = 2$

2. Solve the following equations using matrix method:

i. $4x + 5y = 2$

ii. $3x + \frac{4}{y} = 10$

iii. $2x + 3y = 0$

$-2x + \frac{3}{y} = -1$

Long Questions:

1. Solve the following equations using Cramer's rule and inverse matrix method:

$3x + 5y = 2$

$2x - 3z = -7$

$4y + 2z = 2$

2. Solve the following equation using Row-equivalent matrix method:

i. $9y - 5x = 3$, $x + z = 1$, $z + 2y = 2$

ii. $2x - y + 4z = -3$, $x - 4z = 5$, $6x - y + 2z = 10$

Limit: (Short questions)

1. Find limiting value of following functions:

a. $\lim\limits_{x\to a} \frac{\sqrt{3}-\sqrt{2x+a}}{2(x-a)}$

b. $\lim\limits_{x\to\infty} (\sqrt{x-a}-\sqrt{bx})$

c. $\lim\limits_{x\to\infty} \sqrt{x}\ (\sqrt{x}-\sqrt{x-a})$

d. $\lim\limits_{x\to z} \frac{x-\sqrt{8-x^2}}{\sqrt{x^2+12}-4}$

e. $\lim\limits_{x\to 0} \frac{\sin x^\circ}{x}$

f. $\lim\limits_{x\to 0} x \,.\, \sin\frac{1}{x}$

g. $\lim\limits_{x\to 2} |x-2|$

h. $\lim\limits_{x\to 0} \frac{|x|}{x}$

2. Prove that:

a. $\lim\limits_{x\to 3} \left(\frac{1}{x-3} - \frac{9}{x^3-3x^2}\right) = \frac{2}{3}$

b. $\lim\limits_{x\to 3} \left(\frac{x^2+9}{x^2-9} - \frac{3}{x-3}\right) = \frac{1}{2}$

Long Questions:

1. Prove that $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ where θ is radian.

2. Find limiting value of followings:

a. $\lim\limits_{x\to y} \frac{\cos x-\cos y}{x-y}$

b. $\lim\limits_{\theta\to\frac{\pi}{4}} \frac{\cos\theta-\sin\theta}{\theta-\frac{\pi}{4}}$

c. $\lim\limits_{x\to a} \frac{\sin x-\sin a}{\sqrt{x}-\sqrt{a}}$

d. $\lim\limits_{x\to\theta} \frac{x\sin\theta-\theta\sin x}{x-\theta}$

e. $\lim_{x \to \theta} \dfrac{x \tan \theta - \theta \tan x}{x - \theta}$

Continuity:

1. Test continuity or discontinuity at a given points:

   a. $f(x) = \dfrac{1}{2x}$ at $x = 0$

   b. $f(x) = \dfrac{x^2 - 16}{x - 4}$ at $x = 4$

   c. $f(x) = \dfrac{|x - 2|}{x - 2}$ at $x = 2$

   d. $f(x) = |x|$ at $x = 0$

2. Show that the function $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ is discontinuous at $x = 0$.

   Redefine the function in such a way that it becomes continuous at $x = 0$.

3. Find the value of K for which the function is continuous at indicated point.

   $f(x) = \begin{cases} \dfrac{x^2 - 2x}{x - 2} & \text{if } x \neq 3 \\ K & \text{if } x = 3 \end{cases}$ at $x = 3$.

Polynomial Equation

1. Determine the nature of roots of the equation
   $2x^2 - 3x - 2 = 0$.
2. If the roots of the equation $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$ are equal then $\dfrac{a}{b} = \dfrac{c}{d}$.
3. Find the value of k so that the equation $(3k + 1)x^2 + 2(k + 1) x + k = 0$ may have reciprocal roots.
4. If α and β are the roots of the equation $x^2 - px + q = 0$, find the equation whose roots are $\alpha^2 . \beta^{-1}$ and $\beta^2 . \alpha^{-1}$.
5. Find the condition that the roots of the quadratic equation $ax^2 + bx + c = 0$ may be in the ratio $m : n$.
6. If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, prove that either $p = q$ or $p + q + 1 = 0$.
7. If the quadratic equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, prove that either $a + b + c = 0$ or $a = b = c$.
8. Prove that a quadratic equation $ax^2 + bx + c = 0$ can't have more than two roots.