Uniglobe SS/College

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Mathematics – XI

Logic, Sets and Relations

1. Let p : demand is increasing and q : supply is decreasing. Express each of the following statements into words
   a. $p \wedge \sim q$ b. $\sim(p \wedge q)$
2. Write down the inverse, converse and contrapositive of the compound statement. "If x is a natural number then 2x-1 is an odd number".
3. If p and q are any two statements, prove that
   a. $(p \vee q) \equiv (q \vee p)$
   b. $\sim(p \vee (\sim q)) \equiv (\sim p) \wedge q$
   c. $\sim((\sim p) \wedge q) \equiv p \vee (\sim q)$
4. Construct the truth table of $p \wedge (\sim p \Rightarrow q)$.
5. For any statement p, show that $p \wedge \sim p$ is a contradiction.
6. State and prove De-Morgan's law.
7. If A, B and C are subsets of a universal set U, prove that:
   i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   ii) $A - (B \cup C) = (A-B) \cap (A-C)$
   iii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Relation and Function

8. If A = {1, 2, 3}, find a relation on A satisfying the condition x+y > 4. Is the relation a function? [R = {(2, 3), (3,2), (3,3)}, not a function]
9. Let A ={ 1, 2, 3, 4} and B = {1, 3, 5}. Find a relation from set A to B determined by the condition x>y. [R= {(2, 1), (3,1), (4, 1),(4, 3)}]
10. If A = { x: x= 2n+1, n<5, $n \in N$ } and B = { y: y= 3n-2, n< 4, $n \in N$ }, find $A \cap B$ and $A \cup B$. [$A \cap B$ = {7}, $A \cup B$ = { 1, 3, 4, 5, 7, 9}]
11. Define function. Also define one-one onto and one-one into function. Show that the function f: [1, 4] $\to$ R defined by $f(x) = x^2$ is one-one but not onto.
12. Let f : R $\to$ R and g : R $\to$ R be defined by f(x) = 2x + 1 and g(x) = 3x – 1, find (gof) (x) and (fog) (x).
13. Let a function f : A $\to$ B be defined by $f(x) = \frac{x-1}{x+2}$ with A = {–1, 0, 1, 2, 3, 4) and $B = \left\{-2, 1, -\frac{1}{2}, 0, \frac{1}{2}, \frac{1}{4}, \frac{2}{5}\right\}$. Find the range of f. Is the function f one to one and onto both? If not, how can you make if one to one and onto both?

Matrices and Determinants

14. If $A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$ and B $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, verify that (A + B)' = A' + B'.
15. If $A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, show that A^T. A = 1.
16. If $A = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$, find AA^T.
17. Define symmetric matrix. If $A = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 3 & 4 \\ 5 & 2 & 6 \end{pmatrix}$ find $A + A^T$. Is $A + A^T$ symmetric matrix?
18. Show that:
   i. $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (y-z)(z-x)(x-y)$.
   ii. $\begin{vmatrix} 1 & a^2 & a^2 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} = \begin{vmatrix} bc & a & a^2 \\ ca & b & b^2 \\ ab & c & c^2 \end{vmatrix}$
   iii. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$
   iv. $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$
   v. Without expanding show that $\begin{vmatrix} b & c & b+c \\ c & a & c+a \\ a & b & a+b \end{vmatrix} = 0$

Coordinate Geometry

19. If a line with equation 5x + 6y = 2k together with the coordinate axes form a triangle of area 135 sq. units. Find the value of k. [k = 145]
20. Determine the equation of the straight lines through (1, –4) that makes an angle of 45° with the straight line 2x + 3y + 7 = 0.
   [$m = -5, \frac{1}{5}$, y + 5x = 11, 5y – x + 21 = 0]

21. Find the equation of the sides of an equilateral triangle whose vertex is (–1, 2) and base is y = 0. [$\sqrt{3}x - y + 2 + \sqrt{3} = 0$; $\sqrt{3}x + y + \sqrt{3} - 2 = 0$]

22. Find the angle between the lines whose equations are $y = m_1x + c_1$ and $y = m_2x + c_2$. Also find the condition under which the two lines are (i) parallel, (ii) perpendicular.

Quadratic Equation

23. If the roots of the equation $lx^2 + mx + n = 0$ be in the ratio p : q. Find the value of $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}}$. [$-\frac{m}{\sqrt{ln}}$]

24. If the roots of $lx^2 + mx + n = 0$ be in the ratio 3 : 4. Show that: $12m^2 = 49\,ln$.

25. Form a quadratic equation whose roots are the reciprocals of the roots of $lx^2 + mx + n = 0$. [$nx^2+mx+l=0$]

26. If the quadratic equation $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, then either a + b + c = 0 or a = b = c.

27. If the roots of $(c^2+d^2)x^2 - 2a(ac+bd)x + (a^2+b^2) = 0$ are equal, the prove that $\frac{a}{b} = \frac{c}{d}$

28. If the quadratic equation $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root. Prove that either $p = q$ or $p + q + 1 = 0$

29. The sum of root of the equations $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. Prove that product of the roots is $\frac{-1}{2}(a^2+b^2)$.

30. Prove that quadratic equation cannot have more than two roots.

Calculus

31. Evaluate the limits of

a. $\lim_{x\to 2} \frac{x^2 - 5x + 6}{x^2 - x - 2}$ [Ans: -1/3]

b. $\lim_{x\to \infty} \frac{5x^2 + 2x - 7}{3x^2 + 5x + 2}$ [Ans: 5/3]

c. $\lim_{x\to \infty} (\sqrt{x} - \sqrt{x-3})$ [Ans: 0]

d. $\lim_{x\to \infty} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{x-1}}$ [Ans: 1/2]

e. $\lim_{x\to \infty} \sqrt{x}\left(\sqrt{x} - \sqrt{x-3}\right)$ [Ans: 3/2]

32. Find the limits of

a. $\lim_{x\to a} \frac{\sqrt{3x} - \sqrt{2x+a}}{2(x-a)}$ [Ans: $\frac{1}{4\sqrt{3a}}$]

b. $\lim_{x\to 2} \frac{x - \sqrt{8 - x^2}}{\sqrt{x^2 + 12} - 4}$ [Ans: 4]

c. $\lim_{x\to 1} \frac{\sqrt{2x} - \sqrt{3 - x^2}}{x - 1}$ [Ans: $\sqrt{2}$]

33. Evaluate the limits if exists.

a. $f(x) = \begin{cases} 2x^2 + 1 & \text{for } x \le 2 \\ 4x + 1 & \text{for } x > 2 \end{cases}$ at x = 2 [Ans: 9]

b. $f(x) = \begin{cases} 3x + 2 & \text{for } x \ge 1 \\ 2x & \text{for } x < 1 \end{cases}$ at x = 1 [Limit does not exist]

34. Evaluate:

a. $\lim_{x\to \theta} \frac{x\cot\theta - \theta\cot x}{x - \theta}$ [Ans: $\cot\theta + \theta\operatorname{cosec}^2\theta$]

b. $\lim_{x\to y} \frac{x\cos y - y\cos x}{x - y}$ [Ans: cos y + y sin y]

c. $\lim_{x\to y} \frac{\tan x - \tan y}{x - y}$ [Ans: $\sec^2 y$]

35. A function f(x) defined below:

a. $f(x) = \begin{cases} \frac{2x^2 - 18}{x - 3} & \text{for } x \ne 3 \\ a & \text{for } x = 3 \end{cases}$

is continuous at x = 3, find a. [Ans: 12]

b. $f(x) = \begin{cases} 3ax + b & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax - 2b & \text{if } x < 1 \end{cases}$

is continuous at x = 1, find a and b. [Ans: a=3, b=2]

36. Discuss the continuity of functions at the points specified.

a. $f(x) = \begin{cases} 2x + 1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$ at x = 1 [Ans: discontinuous]

b. $f(x) = \begin{cases} \frac{\sin 2x}{3x} & \text{for } x \ne 0 \\ \frac{3}{2} & \text{for } x = 0 \end{cases}$ at x = 0 [Ans: discontinuous]

Wish you a very happy Dashain and Tihar !