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Mathematics – XII

Part I: ALGEBRA

1. a. If ${}^nP_r = 720$ and ${}^nC_r = 120$, find r. [Ans :3]

   b. If ${}^{16}C_r = {}^{16}C_{r+4}$, find the value of ${}^rC_3$. [Ans :20]

2. a. In how many ways 5 people can be seated in a round table? [Ans:24]

   b. In how many ways can 10 people be seated around a table if two people insist on sitting next to each other? [Ans :80640]

3. a. From 15 persons in how many ways can a selection of 6 be made if 1 particular person is always included? [Ans :2002]

   b. From 10 persons in how many ways can a selection of 4 be made if 1 particular person is always excluded? [Ans :126]

4. How many numbers are there between 100 and 1000 in which all the digits are distinct ? [Ans :648]

5. How many numbers are there between 100 and 1000 such that every digits is either 2 or 9. [Ans :8]

6. How many numbers having at least three different digits can be formed from the integers 3, 4, 5, 6, 7? [Ans :300]

7. In how many ways the letters of the word "PRECARIOOS" be arranged so that all vowels are together? [Ans :21600]

8. Show that the number of ways in which the letters of word "arrange" can be arranged so that two r's do not come together is 960?

9. A committee of five is to be formed from six boys and five girls. In how many ways can this be done so that the committee has majority of girls? [Ans: 181]

10. Find the term independent of x in the expansion of

    a. $\left(x^2 + \frac{1}{x}\right)^{12}$ [Ans: 495] b. $\left(x^2 + \frac{1}{x}\right)^{15}$ [Ans:3003]

11. a. Find the coefficient of x^6 in the expansion of $\left(x^2 + \frac{1}{x}\right)^{15}$. [Ans:6435]

    b. Find the coefficient of x^5 in the expansion of $\left(x + \frac{1}{2x}\right)^7$.[Ans: $\frac{7}{2^7}$]

12. State the Binomial theorem. In the expansion of $(1 + x)^n$ prove that the sum of the coefficients of the odd term is equal to the sum of coefficients of even terms and each equal to $2^n - 1$.

13. Show that middle term in the expansion of $\left(x - \frac{1}{x}\right)^{2n}$ is $\frac{1.3.5 \ldots (2n-1)}{n!}(-2)^n$.

Part II: CALCULUS

14. From the first principle, find the derivatives of:

    a) $\cos^{-1}(x)$ b) $\tan^{-1} x$. c) $\sin(\log x)$

    d) $\log(\tan x)$ e) $e^{\tan x}$ f) $\sin x^2$

    Answers: a) $-\frac{1}{\sqrt{1-x^2}}$ b) $\frac{1}{1+x^2}$ c) $\frac{1}{x}\cos(\log x)$

    d) $\frac{\sec^2 x}{\tan x}$ e) $\sec^2 x.\, e^{\tan x}$ f) $2x.\cos x^2$

15. a. If $x^y = e^{x-y}$ prove that: $\frac{dy}{dx} = (\log x)\ (\log ex)^{-2}$

    b. If $x = \frac{\sin(a+y)}{\sin y}$, Prove that; $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

16. a. Find the point on the curve $y = 3x^2 + 4x - 5$ where the tangent is parallel to the line $16x + 2y = 3$. [Ans: (-2, -1)]

    b. Find the equation of the tangents to the curve $y = (x - 1)(x - 2)$ at the points where the curve meets the x- axis. [Ans: x-y =2]

    c. At what angle does the curve $y(1 + x) = x$ cut the x-axis ? [Ans:45^0]

17. Using L-Hospital's rule. Evaluate the following:

    a. $\lim_{x\to\infty} \frac{3x^2 + 4x + 7}{2x^2 + 5x + 8}$ b. $\lim_{x\to 0} \frac{xe^x - \log(x+1)}{x^2}$

    c. $\lim_{x\to 0} \frac{\log\tan x}{\log x}$ d. $\lim_{x\to 0} \frac{x - \sin x\cos x}{x^3}$

Answers: a) 3/2 b) 3/2 c) 1 d) 2/3

18. State Rolle's theorem. Give its geometrical interpretation. Verify the Rolle's theorem for the function $f(x) = \sqrt{1 - x^2}$ in the interval $-1 \le x \le 1$. [Ans: 0]

19. State Lagrange's Mean value theorem. Give its geometrical interpretation. Verify the Lagrange's mean value theorem for the function $f(x) = (x - 1)(x - 2)(x - 3)$ in [1, 4]. [Ans: 3]

Part III: CO-ORDINATE GEOMETRY

20. Find equation of a parabola with focus at (3, 2) and directrix x+y-3 =0.

[Ans: $x^2+y^2-2xy-6x-2y+17 =0$]

21. Find the equation and point of contact of the tangent to a parabola $y^2 = 8x$ drawn from the point (3, 5). [Ans: 3y=2x+9, y=x+2, (9/2, 6), (2, 4)]

22. Find the equation of tangent to the parabola $y^2=16x$ inclined at an angle 60^0 with x-axis. Also find the point of contact. [Ans: $3x-\sqrt{3}y+4=0$, $(4/3, \frac{8}{\sqrt{3}})$]

23. Show that the normal to the parabola $y^2=16x$ at (1, 4) meets the parabola again at (81, -36).

24. Prove that a line y=mx+c may be normal to the parabola $y^2=4ax$ if $2am+am^3+c=0$.

25. Show that the tangents to the parabola $y^2=12x$ and $x^2=12y$ at (3, 6) and (-6, 3) respectively are at right angles.

26. Prove that the tangents and normal at the ends of the latus rectum of a parabola form sides of a square.

Part IV: COMPUTATIONAL MATHEMATICS

27. Solve the following linear systems by the Gauss elimination method:

a) $\begin{aligned} -x+y &= 9 \\ x-3y &= 5 \end{aligned}$ b) $\begin{aligned} x+y+z &= 6 \\ x-y+z &= 2 \\ 2x+y-z &= 1 \end{aligned}$ c) $\begin{aligned} x+5y+2z &= 6 \\ 2x+3y+z &= 0 \\ 3x-y+z &= -2 \end{aligned}$

Answers: a) (-16, -7) b) (1, 2, 3) c) (-2, 0, 4)

28. Use the Gauss-Siedel method to solve the systems.

a) $\begin{aligned} 3x+y &= 5 \\ x-3y &= 5 \end{aligned}$ b) $\begin{aligned} 2x+3y &= 12 \\ 3x+2y &= 13 \end{aligned}$ c) $\begin{aligned} 4x-y+z &= 8 \\ 2x+5y+2z &= 3 \\ x+2y+4z &= 11 \end{aligned}$

Answers: a) (2, -1) b) (3, 2) c) (1, -1, 3)

29. Solve the following systems of equations by matrix inversion method:

a. $\begin{aligned} x + y + z &= 6 \\ x - y + z &= 2 \\ x + y - z &= 0 \end{aligned}$ b. $\begin{aligned} 2x - y + z &= -2 \\ x + y - 2z &= -9 \\ x + 2y + z &= 9 \end{aligned}$

Answers: a) (1, 2, 3) b) (-2, 3, 5)

30. Compute an approximate value of $\int_0^1 (1 + x^2)^{-1}\, dx$ by using the composite trapezoid rule with three points. Then compare with the actual value of the integral. Next, determine the error formula and numerically verify an upper bound on it.

31. Find an approximate value of the integral of $f(x) = x^2$ from x = 0 to x = 6 using (a) Composite trapezoidal rule (b) Composite Simpson's 1/3 rule with n = 6. Compare the results with the actual value of the integral.

32. Apply the successive bisection method to find the root of the equation $x^3 - x - 4 = 0$ lying between 1 and 2 correct to two places of decimal. **Ans: 1.79**

33. Using bisection method, find the root of the equation $2x^3 - 5x + 2 = 0$, $x \in (1, 2)$ with error less than 10^{-2}. **Ans: 1.32**

34. Derive the iterative formula for Newton-Raphson method. Using Newton Raphson method, find a positive root of $x^3 + 3x - 5 = 0$ lying between 1 and 2 correct to three places of decimals. **Ans: 1.154**

35. Use Newton–Raphson method to find the solution of the equation $x^3 + x - 1 = 0$ in the interval [0, 1], accurate to within 10^{-4}. **Ans: 0.68234**